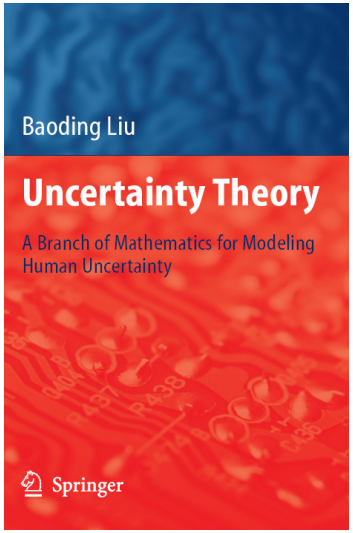
Why Uncertainty?
Uncertainty Theory
Uncertain Programming
Uncertain Risk Analysis
Uncertain Reliability Analysis
Uncertain Process
Uncertain Calculus
Uncertain Differential Equation
Uncertain Finance
Uncertain Logic
Uncertain Entailment
Uncertain Set Theory
Uncertain Inference
Uncertain Control
Uncertain Statistics



Law of Truth Conservation
Maximum Uncertainty Principle
Evolution of Measures
Uncertainty vs Probability
Uncertainty vs Fuzziness
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Uncertainty Theory

Uncertainty theory is a branch of mathematics based on normality, monotonicity, self-duality, countable subadditivity, and product measure axioms.

Axiom 1. (Normality) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2. (Monotonicity) $\mathcal{M}\{\Lambda_1\} \leq \mathcal{M}\{\Lambda_2\}$ whenever $\Lambda_1 \subset \Lambda_2$.

Axiom 3. (Self-Duality) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 4. (Countable Subadditivity) $\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$.

Axiom 5. (Product Measure Axiom) $\mathcal{M}\left\{\prod_{k=1}^{n} \Lambda_k\right\} = \min_{1\leq k\leq n} \mathcal{M}_k\{\Lambda_k\}$.

Liu B., Uncertainty Theory, 2nd ed., Springer-Verlag, Berlin, 2007.

Uncertain Programming

Uncertain programming is a type of mathematical programming involving uncertain variables.

$$\begin{cases} \max\limits_{\mathbf{x}} \max\limits_{\overline{f}} \overline{f} \\ \text{subject to:} \\ \quad \mathcal{M}\{f(\mathbf{x},\boldsymbol{\xi}) \geq \overline{f}\} \geq \beta \\ \quad \mathcal{M}\{g(\mathbf{x},\boldsymbol{\xi}) \geq 0\} \geq \alpha \end{cases}$$

Liu B., Theory and Practice of Uncertain Programming, 2nd ed., Springer-Verlag, Berlin, 2009.

Uncertain Risk Analysis

Uncertain risk analysis is a tool to quantify risk via uncertainty theory.

$$Risk = \mathcal{M}\{L(\xi_1, \xi_2, \cdots, \xi_n) \leq 0\}$$

Liu B, Uncertain risk analysis and uncertain reliability analysis, *Journal of Uncertain Systems*, Vol.4, No.3, 163-170, 2010.

Uncertain Reliability Analysis

Uncertain reliability analysis is a tool to deal with system reliability via uncertainty theory.

$$\textit{Reliability} = \mathcal{M}\{R(\xi_1, \xi_2, \cdots, \xi_n) \geq 0\}$$

Liu B, Uncertain risk analysis and uncertain reliability analysis, *Journal of Uncertain Systems*, Vol.4, No.3, 163-170, 2010.

Uncertain Process

Uncertain process is a sequence of uncertain variables indexed by time or space.

$$X_t : T \times (\Gamma, \mathcal{L}, \mathcal{M}) \to \Re$$

A Sequence of Uncertain Variables indexed by Time

Liu B, Fuzzy process, hybrid process and uncertain process, *Journal of Uncertain Systems*, Vol.2, No.1, 3-16, 2008.

Uncertain Calculus

Uncertain calculus is a branch of mathematics that deals with differentiation and integration of function of uncertain processes.

C_t: Canonical Process

$$\int_a^b X_t \mathrm{d}C_t = \lim_{\Delta \to 0} \sum_{i=1}^{k} X_{t_i} \cdot (C_{t_{i+1}} - C_{t_i})$$

$$\mathrm{d}X_t = \frac{\partial h}{\partial t}(t, C_t)\mathrm{d}t + \frac{\partial h}{\partial c}(t, C_t)\mathrm{d}C_t$$

Liu B, Some research problems in uncertainty theory, *Journal of Uncertain Systems*, Vol.3, No.1, 3-10, 2009.

Uncertain Differential Equation

Uncertain differential equation is a type of differential equation driven by canonical process.

$$\mathrm{d}X_t = f(t, X_t)\mathrm{d}t + g(t, X_t)\mathrm{d}C_t$$

Liu B, Fuzzy process, hybrid process and uncertain process, *Journal of Uncertain Systems*, Vol.2, No.1, 3-16, 2008.

Uncertain Logic

Uncertain logic is a generalization of mathematical logic for dealing with uncertain knowledge via uncertainty theory.

$$T(X) = \mathcal{M}\{X = 1\}$$


Li X and Liu B, Hybrid logic and uncertain logic, *Journal of Uncertain Systems*, Vol.3, No.2, 83-94, 2009.


Zhang XF and Peng ZX, Uncertain predicate logic based on uncertainty theory, http://orsc.edu.cn/online/091204.pdf.

Uncertain Entailment

Uncertain entailment is a methodology for calculating the truth value of an uncertain formula via the maximum uncertainty principle when the truth values of other uncertain formulas are given.

ξ, η, τ: uncertain propositions with unknown truth values

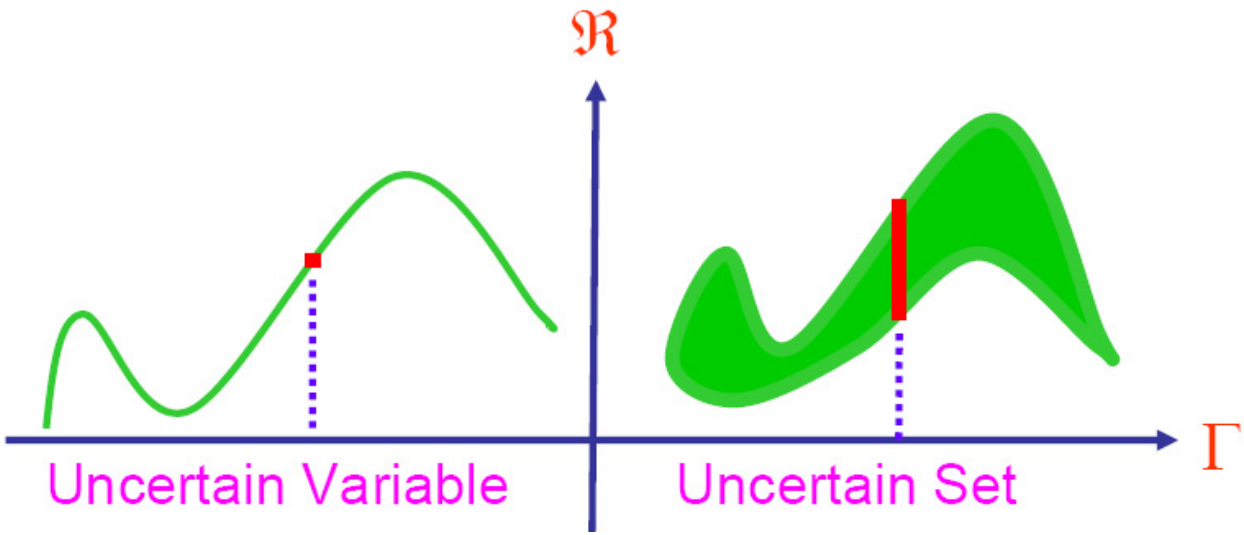
$$T(\xi \to \eta) = 0.7, \qquad T(\eta \to \tau) = 0.8$$

What is $T(\xi \to \tau)$?

Liu B, Uncertain entailment and modus ponens in the framework of uncertain logic, *Journal of Uncertain Systems*, Vol.3, No.4, 243-251, 2009.

Uncertain Set Theory

Uncertain set theory is a generalization of uncertainty theory to the domain of uncertain sets.



Liu B, Uncertain set theory and uncertain inference rule with application to uncertain control, *Journal of Uncertain Systems*, Vol.4, No.2, 83-98, 2010.

Uncertain Inference

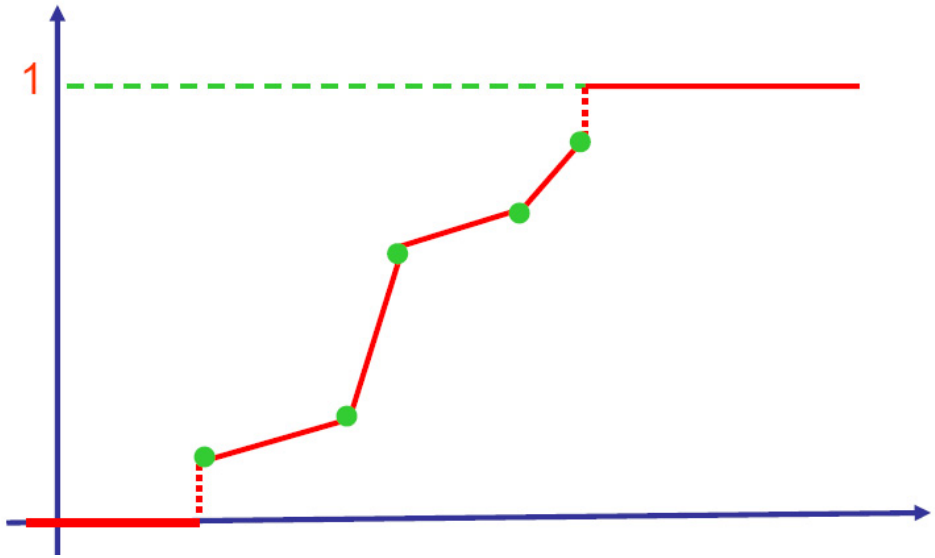
Uncertain inference is a process of deriving consequences from uncertain knowledge or evidence via the tool of conditional uncertain set.

$$\frac{\begin{array}{l}\text{If } \mathbb{X} \text{ is } \xi, \text{ then } \mathbb{Y} \text{ is } \eta \\ \mathbb{X} \text{ is uncertain set } \xi^* \end{array}}{\mathbb{Y} \text{ is conditional } \eta^* = \eta|_{\xi^* \triangleright \xi}}$$

Liu B, Uncertain set theory and uncertain inference rule with application to uncertain control, *Journal of Uncertain Systems*, Vol.4, No.2, 83-98, 2010.

Uncertain Statistics

Uncertain statistics is a methodology for collecting and interpreting expert's experimental data by uncertainty theory.



Liu B, *Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty*, Springer-Verlag, Berlin, 2010.