Based on the optimistic value model of uncertain optimal control model with jump, in this paper, an optimistic value model of uncertain linear quadratic (LQ) optimal control with jump is proposed. Then, the necessary and sufficient condition for the existence of optimal control is obtained. Finally, an enterprise’s inventory problem is given to illustrate usefulness of the proposed model.

Keywords: optimistic value, optimal control, uncertainty, jump, linear quadratic model

1. Introduction

Linear quadratic (LQ) optimal control is one of the most important and fundamental classes of optimal control problems where the system dynamics are linear and the objective functional is quadratic. It was widely used in many fields of modern real life such as production inventory, portfolio selection, engineering, etc. LQ optimal control problem was first studied by Kalman [1] (for deterministic). Extension to stochastic LQ optimal control was first carried out by Wonham [2, 3] and then developed by Bismut [4], Bonsoussan [5], Davis [6], Chen et al. [7], Wu and Zhou [8], Zhou and Li [9]. With the effort of many researchers in the last 50 years, there has been an enormously rich theory on LQ optimal control, deterministic and stochastic alike, and it has been one of the most perfect parts in the optimal control theory and applications.

As we all know, when we study stochastic LQ optimal control problem, to determine a probability distribution, we need a large amount of historical data. But for many events in the real world, no samples are available or the set of the available samples is too small. In this case, we have to invite some domain experts to evaluate their belief degree that each event will occur. Perhaps some people think that personal belief degree is subjective probability or fuzzy concept. However, Liu [10] showed that it is inappropriate because both probability theory and fuzzy set theory may lead to counterintuitive results in this case. To rationally deal with the belief degree, an uncertainty theory was founded by Liu [11] in 2007 and subsequently studied by many scholars. Nowadays, uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty and has been developed in a variety of directions, including uncertain programming, uncertain finance, uncertain statistics, etc.

As an application of uncertainty theory, Zhu [12] proposed and dealt with an expected value model of uncertain optimal control problem without jump in 2010. An equation of optimality as a counterpart of Hamilton-Jacobi-Bellman equation was obtained by applying dynamic programming principle. After that, Kang and Zhu [13], Xu and Zhu [14] discussed uncertain bang-bang optimal control problems for continuous time and for multi-stage systems without jump, respectively. Furthermore, Yan and Zhu [15] investigated the bang-bang optimal control problem for uncertain switched systems. Besides, Chen and Zhu [16] studied the uncertain optimal control problem with time-delay. Yao and Qin [17] proposed an uncertain linear quadratic control model. In order to consider the effects of jumps on the optimal policies, Deng and Zhu [18–21] studied some expected value models of uncertain optimal control problems with jump. Recently, Sheng and Zhu [22] considered making use of the optimistic value criterion to optimize the uncertain objective function in case without jump. In [23], Deng studied the uncertain optimal control model with jump under optimistic value criterion. In this paper, we will further study the optimistic value model of uncertain linear quadratic (LQ) optimal control with jump.

The rest of the paper is organized as follows. In Section 2, an optimistic value model of uncertain optimal control problem with jump and the equation of optimality for the given model are reviewed. In Section 3, an optimistic value model of uncertain LQ optimal control with jump is proposed and necessary and sufficient condition for optimal control is derived. In Section 4, an enterprise’s inventory problem is discussed to illustrate the application of the result. In Section 5, conclusions are made.
2. Optimistic Value Model of Uncertain Optimal Control with Jump

An optimistic value model of uncertain optimal control with jump was considered in Deng [23], and restated as follows

\[
J(t,x) \equiv \sup_{u \in U} \left\{ \int_t^T f(s, X_s, u_s) ds + G(T, X_T) \right\} \sup \{ \alpha \} \text{ subject to}
\]
\[
dX_s = v(s, X_s, u_s) ds + \gamma(s, X_s, u_s) dC_s + \chi(s, X_s, u_s) dV_s, \quad X_t = x
\]

where \(X_s\) is the state variable, \(u_s\) is the control variable subject to a constraint set \(U\). The function \(f: [0, T] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \rightarrow \mathbb{R}\) is the objective function, and \(G: [0, T] \times \mathbb{R} \rightarrow \mathbb{R}\) is the function of terminal reward. \([I_s^T f(s, X_s, u_s) ds + G(T, X_T)]\sup\{ \alpha \}\) denotes the \(\alpha\)-optimalistic value to the uncertain variable in middle bracket. In addition, \(v, \gamma, \chi\) are three functions of time \(s\), state \(X_s\) and control \(u_s\). \(C_t\) is a canonical process and \(V_t\) is an uncertain jump process with parameters \(r_1\) and \(r_2\) \((0 < r_1 < r_2 < 1)\), where \(C_t\) and \(V_t\) are independent. \(\alpha \in (0, 1)\) denotes confidence level.

**Theorem 1:** (Deng [23]) [Equation of optimality] Let \(J(t,x)\) be twice differentiable on \([0, T] \times \mathbb{R}\). Then we have

\[
-J_t = \sup_{u \in U} \left\{ f + J_x v + \frac{\sqrt{\alpha}}{\pi} \ln \frac{1 - \alpha}{\alpha} [J_x \gamma + k J_x \chi] \right\},
\]
where \(J_t\) and \(J_x\) are the partial derivatives of the function \(J(t,x)\) in \(t\) and \(x\), respectively, and \(f, v, \gamma, \chi, J_t, J_x\) denote \(f(t, x, u), v(t, x, u), \gamma(t, x, u), J_t(t, x), J_x(t, x), J_{xx}(t, x)\), respectively, and

1. if \(J_t \chi \geq 0\), then \(k = 1 - \frac{a}{2(1 - r_2)} \left(0 < \alpha < 1 - r_2\right)\) or \(k = \frac{a}{2r_1(1 - r_1)} \left(1 - r_1 \leq \alpha < 1\right)\);
2. if \(J_t \chi < 0\), then \(k = \frac{a}{2r_1} \left(0 < \alpha \leq r_1\right)\) or \(k = \frac{1 - a}{2(1 - r_2)} \left(r_2 \leq \alpha < 1\right)\).

3. Optimistic Value Model of Uncertain LQ Optimal Control with Jump

In this section, we propose an optimistic value model of uncertain LQ optimal control with jump as follows

\[
J(0,x) = \min_{u} \left\{ \int_0^T \left( A_1 X_t^2 + A_2 u_t^2 + A_3 X_t u_t \right) dt + G_T X_T^2 \right\} \sup \{ \alpha \} \text{ subject to}
\]
\[
dX_t = (a_1 X_t + a_2 u_t + a_3) dt + (b_1 X_t + b_2 u_t + b_3) dC_t + (c_1 X_t + c_2 u_t + c_3) dV_t, \quad X_0 = x_0.
\]

The physical meaning of the quadratic performance index in model (3) is to make the dynamic error and energy consumption of the system in control process, as well as the steady-state error at the end of the control to achieve mini-

\[
-\frac{A_3}{2A_2} x + A_3 + (a_2 + \tilde{k} b_2 + k c_3) [P(t)x + Q(t)]
\]

where \(x\) is the state of the state variable \(X_t\) at time \(t\) obtained by applying the optimal control \(u^*_t\), the function \(P(t)\) satisfies the following Riccati differential equation and boundary condition

\[
dP(t) = \frac{(a_2 + \tilde{k} b_2 + k c_3)^2}{2A_2} P(t) + \frac{A_3 (a_2 + \tilde{k} b_2 + k c_3)}{2A_2} \left[a_1 + \tilde{k} b_1 + k c_1 \right] P(t) + \frac{A_3^2}{2A_2} - 2A_1,
\]

and the function \(Q(t)\) satisfies the following differential equation and boundary condition

\[
-\frac{(a_2 + \tilde{k} b_2 + k c_3)^2}{2A_2} P(t) + \frac{A_3 (a_2 + \tilde{k} b_2 + k c_3)}{2A_2} \left[a_1 + \tilde{k} b_1 + k c_1 \right] Q(t) + \frac{A_3^2}{2A_2} - 2A_4
\]

The optimal value is

\[
J(0,x_0) = \frac{1}{2} \left( x_0^2 [P(0)] + x_0 Q(0) + R(0) \right).
\]
where \( R(t) \) satisfies
\[
\frac{dR(t)}{dt} = -\frac{(a_2 + \bar{k}b_2 + k_c)^2}{4A_2} Q^2(t) \\
- \left[ A_5 \left( a_2 + \bar{k}b_2 + k_c \right) - \frac{(a_3 + \bar{k}b_3 + k_c)}{2A_2} \right] Q(t) \\
- \frac{A_5^2}{4A_2} + A_6, \quad R(T) = 0.
\]
and

(1) if \( J_x \geq 0, J_y \geq 0 \), then
\[
k = \frac{\sqrt{\pi} \ln \frac{1-\alpha}{\alpha}}{2(a-\bar{k})}, \quad k = \frac{\sqrt{\pi} \ln \frac{1-\alpha}{\alpha}}{2(a-\bar{k})}.
\]

(2) if \( J_x \geq 0, J_y < 0 \), then
\[
k = \frac{\sqrt{\pi} \ln \frac{1-\alpha}{\alpha}}{2(a-\bar{k})}.
\]

(3) if \( J_x \leq 0, J_y \geq 0 \), then
\[
k = \frac{\sqrt{\pi} \ln \frac{1-\alpha}{\alpha}}{2(a-\bar{k})}.
\]

(4) if \( J_x < 0, J_y < 0 \), then
\[
k = \frac{\sqrt{\pi} \ln \frac{1-\alpha}{\alpha}}{2(a-\bar{k})}.
\]

Proof: We only prove that conclusion holds under satisfying the condition (1), the other part of the theorem can be proved similarly.

The necessity will be proved first. If \( J_x \geq 0, J_y \geq 0 \) and \( 0 < \alpha < 1 - r_2 \) for given \( \alpha \), it follows from the equation of optimality (2) that
\[
-J_t = \min_u \left[ A_1 x^2 + A_2 u_t^2 + A_3 x u_t + A_4 x + A_5 u_t + A_6 \\
+ (a_1 x + a_2 u_t + a_3) J_x \\
+ \sqrt{\frac{\pi}{2}} (b_1 x + b_2 u_t + b_3) J_x \ln \frac{1-\alpha}{\alpha} \\
+ \left( 1 - \frac{\alpha}{2(1-r_2)} \right) (c_1 x + c_2 u_t + c_3) J_x \right]
\]

where \( L(u) \) represents the term in the braces. Letting \( \partial L(u)/\partial u = 0 \) yields
\[
2A_2 u_t + A_3 x + A_5 + a_2 J_x + \sqrt{\frac{\pi}{2}} b_2 J_x \ln \frac{1-\alpha}{\alpha} \\
+ \left( 1 - \frac{\alpha}{2(1-r_2)} \right) c_2 J_x = 0
\]
or
\[
u_t = -\frac{A_3 x + A_5 + (a_2 + \sqrt{\frac{\pi}{2}} b_2 \ln \frac{1-\alpha}{\alpha}) J_x}{2A_2} \\
- \frac{\left( 1 - \frac{\alpha}{2(1-r_2)} \right) c_2 J_x}{2A_2}.
\]
Substituting \( J_x, J_{ix}, J_{ixx} \) and Eq. (13) into Eq. (12) yields that
\[
\frac{d\varphi(t)}{dt} - \frac{dQ(t)}{dt} = - \left( \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right)^2 P^2(t)
\]
+ \frac{a_2}{2a_2 - 2a_1} \left[ \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right]^2 P(t)
+ \frac{a_2}{2a_2} \left[ \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right] P(t)
- \frac{a_2}{2a_2} \left[ \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right] P(t)
- A_3 A_5 \left[ \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right]
- A_4.
\]
Since \( J_x(T, x) = 2G_T x \), then
\[
J_x(T, x(T)) = 2G_T x(T).
\]
At the same time, we have
\[
J_x(T, x(T)) = P(T) x(T) + Q(T).
\]
It follows from Eqs. (15) and (16) that
\[
P(T) = 2G_T, \quad Q(T) = 0.
\]
By Eqs. (14) and (17), ordinary differential Eqs. (5) and (6) can be obtained. The necessity condition of the theorem is proved under satisfying condition \( J_x \geq 0, J_y \geq 0 \) and \( 0 < a < 1 - r_2 \). If \( J_x \geq 0, J_y \geq 0 \) and \( 1 - r_2 \leq \alpha < 1 - r_1 \) or \( J_x \geq 0, J_y \geq 0 \) and \( 1 - r_1 \leq \alpha < 1 \), the conclusion can be proved similarly. Thus, the necessity condition of the theorem is proved under satisfying the condition (1).

Next we will verify the sufficient condition of the theorem. We conjecture that \( J(t, x) = \frac{1}{2} P(t) x^2 + Q(t) x + R(t) \). Suppose that \( u^*_t, P(t), Q(t), R(t) \) satisfy Eqs. (4), (5), (6), (7), respectively, and then substituting \( J_i = \frac{1}{2} \frac{d\varphi(t)}{dt} x^2 + \frac{dQ(t)}{dt} x + \frac{dR(t)}{dt} \), \( J_i = P(t) x + Q(t) \) and \( u^*_t \) into the equation of the optimality (8) yields that
\[
J_x + A_1 x^2 + A_2 u^*_t + A_3 u^*_t + A_4 + A_5 u^*_t + A_6
+ \left( \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right)^2 P^2(t)
= \frac{1}{2} \left( \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right)^2 P(t)
+ \left( \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right) P(t)
- \left( \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right) P(t)
- A_3 A_5 \left( \frac{a_0 + \sqrt{\frac{3}{\pi}} a_1 b_2 \ln \frac{1-a}{a} + \left( 1 - \frac{a}{2 \ln(1-r)} \right) c_2}{2a_2} \right)
- A_4.
\]
Therefore, we know that \( u^*_t \) is a solution of Eq. (8). Because objective function is convex, Eq. (8) produces a minimum. That means that \( u^*_t \) provided by Eq. (4) is an optimal control. At the same time we also get the optimal value
\[
J(0, x_0) = \frac{1}{2} x_0^2 P(0) + x_0 Q(0) + R(0).
\]
Thus, the sufficient condition of the theorem is proved under satisfying condition \( J_x \geq 0, J_y \geq 0 \) and \( 0 < a < 1 - r_2 \). Therefore, we know that \( u^*_t \) is a solution of Eq. (8). Because objective function is convex, Eq. (8) produces a minimum. That means that \( u^*_t \) provided by Eq. (4) is an optimal control. At the same time we also get the optimal value
\[
J(0, x_0) = \frac{1}{2} x_0^2 P(0) + x_0 Q(0) + R(0).
\]
Thus, the sufficient condition of the theorem is proved under satisfying condition \( J_x \geq 0, J_y \geq 0 \) and \( 0 < a < 1 - r_2 \). Therefore, we know that \( u^*_t \) is a solution of Eq. (8). Because objective function is convex, Eq. (8) produces a minimum. That means that \( u^*_t \) provided by Eq. (4) is an optimal control. At the same time we also get the optimal value
\[
J(0, x_0) = \frac{1}{2} x_0^2 P(0) + x_0 Q(0) + R(0).
\]
can prove the sufficient condition of the theorem under satisfying the conditions (2), (3) and (4). The theorem is proved.

4. Enterprize’s Inventory Problem

Enterprize’s inventory problem is a key problem in supply chain. Many manufacturing enterprises use a production-inventory system to adjust production rate to satisfy consumer demand for the product. Such a system consists of a manufacturing factory producing homogeneous goods and an inventory warehouse which stores finished products which are manufactured but not immediately sold. Once a product is made and put into inventory, it incurs two kinds of inventory holding costs including costs of physically storing the product and opportunity cost of having the firm’s money invested or tied up in the unsold inventory. The inventory optimization problem is to balance the benefits of production smoothing versus the costs of holding inventory.

Applications of optimization methods to production and inventory problems can be dated back to the classical static EOQ (Economic Order Quantity) model of Harris [24]. One of the earliest dynamic control models in this area was the linear-quadratic problem, see Simon [25], Holt et al. [26], Huang et al. [27] and the references therein. Since Merton [28] firstly applied stochastic inventory problem has been thoroughly studied and achieved fruitful results. How-

ventory model as follows:

\[
J(0,i_0) = \min_{L_i} \left\{ \int_0^T \left[ \frac{h}{2} (U_t - I_t)^2 + \frac{c}{2} (U_t - V_t)^2 \right] dt \right. \\
\left. + G_T I_t^2 \right\}_{\text{sup}} (\alpha) \tag{18}
\]

subject to

\[
dI_t = (U_t - S) dt + \sigma_1 dC_t + \sigma_2 dV_t, \quad I_0 = i_0,
\]

where \( L_i \) and \( U_t \) denote, respectively, enterprize’s inventory level (state variable) and the production rate(control variable) at time \( t \), \( I_t \) and \( U_t \) are the inventory goal level and production goal level, respectively. \( S \) denotes the constant sales rate, \( i_0 \) is the initial inventory level, \( h \) and \( c \) represent, respectively, the inventory holding cost coefficient and the production cost coefficient. \( T \) represents the length of planning period. \( G_T \) represents the salvage value per unit of inventory at time \( T \). \( C_t \) and \( V_t \) denote, respectively, independent canonical process and uncertain \( V_t \)-jump process. \( \sigma_1 \) and \( \sigma_2 \) are the constant diffusion coefficient of volatility and the constant jump coefficient of volatility, respectively.

Comparing the model (18) with (3), we know that the parameters of the model (3) are \( A_1 = \frac{1}{2} h, A_2 = \frac{1}{2} c, A_3 = 0, A_4 = -\hat{h}, A_5 = -\hat{c}, A_6 = \frac{1}{2} (\hat{h}^2 + c\hat{U}^2); a_1 = b_1 = b_2 = c_1 = c_2 = 0, a_2 = 1, a_3 = -S, b_3 = \sigma_1, c_3 = \sigma_2\), respectively. Thus, by Theorem 2, we get the optimal control of Eq. (18)

\[
U_t^* = \hat{U} - \frac{P(t) I_t + Q(t)}{c}, \quad \ldots \ldots \ldots \ldots \ldots (19)
\]

and the function \( P(t), Q(t) \) satisfy the following differential equations and boundary conditions

\[
\frac{dP(t)}{dt} = -\frac{1}{c} P^2(t) - h, \quad P(T) = 2G_T, \quad \ldots \ldots \ldots (20)
\]

\[
\frac{dQ(t)}{dt} = \frac{1}{c} P(t) Q(t) + (S - \hat{U} - \hat{k}\sigma_1 - k\sigma_2) P(t) + h\hat{U}, \quad Q(T) = 0, \quad \ldots \ldots \ldots \ldots \ldots \ldots (21)
\]

The optimal value is

\[
J(0,i_0) = \frac{1}{2} \hat{h}^2 P(0) + i_0 Q(0) + R(0),
\]

where \( R(t) \) satisfies

\[
\frac{dR(t)}{dt} = -\frac{1}{2c} Q^2(t) - (S - \hat{U} - \hat{k}\sigma_1 - k\sigma_2) Q(t) + \frac{1}{2} \hat{h}^2, \quad R(T) = 0, \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (22)
\]

Solving Eq. (20), a differential equations of separable variables, we can easily get

\[
P(t) = \frac{\sqrt{ch} \left( e^{\frac{c}{2} \sqrt{ch} T + mc^2 t^{1/2}} \right)}{e^{\frac{c}{2} \sqrt{ch} T - mc^2 t^{1/2}}},
\]

where \( m = \frac{2G_T - \sqrt{c} h}{2G_T + \sqrt{c} h} \).

Let \( n = S - \hat{U} - \hat{k}\sigma_1 - k\sigma_2 \) and \( Q_1(t) = Q(t) + cn \), then
Eq. (21) can be simply expressed as
\[
d Q_1(t) \over dt - \frac{1}{c} P(t) Q_1(t) = h t, \quad Q_1(T) = c n. \quad (23)
\]
Solving first order linear differential Eq. (23), we have
\[
Q_1(t) = \frac{\sqrt{h c}}{t} \left( m e^2 \sqrt{b_1} + e^2 \sqrt{b_2} \right) + p e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
where \( p = (m - 1) c n - (m + 1) \sqrt{h c} T \).
Then
\[
Q_1(t) = \frac{\sqrt{h c}}{t} \left( m e^2 \sqrt{b_1} + e^2 \sqrt{b_2} \right) + p e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
\[
d Q_1(t) \over dt - \frac{1}{2} (c n^2 + h F^2) \quad . \quad . \quad . \quad (25)
\]
Integrating to \( Q_1(t) \), we derive
\[
\int_t^T Q_1^2(s) ds = \frac{\sqrt{h c}}{t} \left( m e^2 \sqrt{b_1} + e^2 \sqrt{b_2} \right) + p e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + c^2 F^2 (T - t)
\]
\[
- \frac{c^2 F^2}{m} (m + 1) + 2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
Therefore
\[
R(t) = \frac{1}{2c} \left[ \int_t^T Q_1^2(s) ds - \frac{1}{2} (c n^2 + h F^2) \right]
\]
\[
= \frac{\sqrt{h c}}{t} \left( m e^2 \sqrt{b_1} + e^2 \sqrt{b_2} \right) + p e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
\[
+ \frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
\[
- \frac{2 c^2 F^2}{m} (m + 1) + 2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{1}{2} \sqrt{h c} (m + 1) + p e^2 e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
\[
\frac{1}{2} \sqrt{h c} (m + 1) + p e^2 e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
It follows from Eq. (19) that the optimal control is
\[
U^*_r = \hat{U} + n - \frac{1}{c} \left[ \sqrt{h c} (e^2 \sqrt{b_1} + m e^2 \sqrt{b_2}) + p e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
\[
+ \sqrt{h c} (m e^2 \sqrt{b_1} + e^2 \sqrt{b_2}) + p e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
\[
\frac{1}{2} \sqrt{h c} (m + 1) + p e^2 e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
\[
\frac{1}{2} \sqrt{h c} (m + 1) + p e^2 e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
\[
\frac{1}{2} \sqrt{h c} (m + 1) + p e^2 e \sqrt{\frac{b_1}{t}}
\]
\[
\frac{2 p c e^2 \sqrt{T} e \sqrt{\frac{b_1}{t}}}{m e^2 \sqrt{b_1} - e^2 \sqrt{b_2}} + \frac{1}{2} \left( c^2 F^2 - c n^2 - h F^2 \right) (T - t)
\]
In order to illustrate the difference between our inventory model and stochastic inventory model, next we make a comparison for two models.
Comparison of results of two models is shown in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertain inventory model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^*_r )</td>
<td>( \hat{U} + \sqrt{h c (m + 1)} (I - T) + \frac{p e^2}{c} \sqrt{T} e \sqrt{\frac{b_1}{t}} )</td>
</tr>
<tr>
<td>( P(t) )</td>
<td>( \frac{\sqrt{h c} (1 + m)}{m} )</td>
</tr>
<tr>
<td>( Q(t) )</td>
<td>( \sqrt{h c (m + 1)} + \frac{p e^2}{c} \sqrt{T} e \sqrt{\frac{b_1}{t}} )</td>
</tr>
<tr>
<td>( R(t) )</td>
<td>( \frac{g}{c} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Stochastic inventory model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^*_r )</td>
<td>( \hat{U} + \sqrt{h c (m + 1)} (I - T) - \frac{\hat{p} e^2}{c} \sqrt{T} e \sqrt{\frac{b_1}{t}} + \frac{\bar{n}}{m} )</td>
</tr>
<tr>
<td>( P(t) )</td>
<td>( \frac{\sqrt{h c} (1 + m)}{m} )</td>
</tr>
<tr>
<td>( Q(t) )</td>
<td>( \sqrt{h c (m + 1)} + \frac{\hat{p} e^2}{c} \sqrt{T} e \sqrt{\frac{b_1}{t}} - \frac{\bar{n}}{m} )</td>
</tr>
<tr>
<td>( R(t) )</td>
<td>( \frac{\bar{g} e^2 - c n^2 - h F^2}{2} (T - t) + \frac{c a^2}{4} \ln \frac{m - 1}{m - y^2} )</td>
</tr>
</tbody>
</table>

Table 1. Comparison of results of two inventory models.
Table 2. The results of the two cases.

<table>
<thead>
<tr>
<th>Parameters /Variable</th>
<th>if $J_1 \geq 0$</th>
<th>if $J_1 &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{k}$</td>
<td>$-1.2114$</td>
<td>$1.2114$</td>
</tr>
<tr>
<td>$k$</td>
<td>$0.1667$</td>
<td>$0.8750$</td>
</tr>
<tr>
<td>$n$</td>
<td>$-9.8521$</td>
<td>$-15$</td>
</tr>
<tr>
<td>$P(t)$</td>
<td>$3 + e^{2(t-8)}$</td>
<td>$3 - e^{2(t-8)}$</td>
</tr>
<tr>
<td>$Q(t)$</td>
<td>$15e^{2(t-8)} + 45 - 40.8357e^{t - 8}$</td>
<td>$15 + 9.8521$</td>
</tr>
</tbody>
</table>

$1 - r_1 = 0.7, 1 - r_2 = 0.4$. Since $\gamma = \sigma_1 > 0, \chi = \sigma_2 > 0$, then we only need to consider two cases, namely $J_1 \chi > 0, J_1 \gamma > 0$ and $J_1 \chi < 0, J_1 \gamma < 0$. The results of the two cases are shown in Table 2.

By using Matlab, we find that if $I \geq 5.4133$, then $J_1 = P(t)I + Q(t) \geq 0$ in the first case; but no matter what value $I$ take, the value of $J_1$ is not less than zero in the second case. According to $I = 15$ we know that condition $I \geq 5.4133$ usually can meet. Therefore, $J_1 > 0$ and then $\tilde{k} = -1.2114, k = 0.1667$. It follows from Eq. (28) that the optimal control is

$$U^*_t = \frac{15e^{2(t-8)} + 45 - 40.8357e^{t - 8}}{3 - e^{2(t-8)}} - \frac{3 + e^{2(t-8)}}{3 - e^{2(t-8)}} I + 20.4179.$$ 

5. Conclusion

This paper presented an optimistic value model of uncertain linear quadratic (LQ) optimal control with jump. The necessary and sufficient condition for the existence of optimal control was obtained. As an application of proposed model, we discussed an enterprise’s inventory problem. In future work, we will further study the optimistic value model of uncertain LQ optimal control problem with jump in case of multi-dimension.

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