Stock Loan Valuation under Uncertain Mean-reverting Stock Model

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Abstract
This paper is concerned with valuation of stock loans. The underlying stock price is assumed to follow a mean reverting uncertain differential equation driven by canonical Liu process in this paper. The price formulas of standard stock loan and capped stock loan are derived by using method of uncertain calculus within the framework of uncertainty theory.

Keywords stock loan, mean-reverting, uncertainty theory, uncertain differential equation, uncertain stock model

1 Introduction

Stock loan is different from the traditional loans, and the stocks are employed as the only guarantee for this type of loan. Stock loan is a contract between a borrower and a bank that gives the borrower the right rather than obligation to regain his or her stock at any time before the loan maturity by repaying the

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bank the principal plus the loan interest in the case of a borrower obtains some money from a bank with his or her stock as collateral. This financial product has many advantages for the borrowers, for example, stock loan can be used as a hedge tool against the letting down of stock market, as the price of the stock goes down, the borrower can choose to give up the collateral rather than to regain the stock to avoid the loss from devaluation of the stock. On the other hand, if the stock price goes up, he or she can choose to redeem the stock by repaying the bank the loan amount and the loan interest. Another advantage is reflected in such an situation that the borrower needs money urgently but he or she is unwilling to lose his or her ownership of stocks completely.

The associated research of stock loan was pioneered by Xia and Zhou (2007). They derived a closed form pricing formula of stock loan based on the classical Black-Scholes model by using a purely probabilistic approach. Then Zhang and Zhou (2009) extended their framework to a problem of valuation of stock loans with regime switching model and gave the stock loan pricing formulas for this type of model. Afterwards, the problem of stock loan pricing are investigated by many scholars, including Liang, Wu and Jiang (2010), Wong and Wong (2012), Pascucci, Suarez-Taboada and Vazquez (2013) and Cai and Sun (2014) and so on.

The previous researches on stock loans valuation are all within the framework of probability theory. The stock loans pricing problem were solved by using probabilistic approach based on the assumption that the stock price follows the stochastic differential equations. However, this assumption was challenged by many scholars. Liu (2013) proposed a paradox that gave a convincing explanation to show that using stochastic differential equations to describe stock price is inappropriate. Kahneman and Tversky (1979) showed that the probability itself is not served as the basis of decision makings by investors, and investors usually make a nonlinear transformation of probability as their basis which they based on to make decisions. Liu (2015) expressed the view that human beings usually estimate a much wider range of values than the object actually takes. In real financial practice, investors’ belief degrees usually play an important role in decision makings and affect the financial market performance. The belief degrees behave neither like randomness nor like fuzziness, and it can not be described with probability theory and fuzzy theory. For rationally dealing with human’s belief degrees, Liu (2007) founded uncertainty theory in 2007 and refined it in 2010. For modeling the evolution of phenomena with uncertainty, Liu
gave the concept of uncertain process, and established uncertain calculus to deal with differentiation and integration of uncertain processes.

Different from Black-Scholes framework, Liu (2009) proposed an uncertain stock model in which the stock price is described by an uncertain differential equation. Liu (2009), Chen (2011) and Zhang and Liu (2014) gave the pricing formulas of European option, American option and geometric average Asian option for Liu’s uncertain stock model, respectively. Peng and Yao (2011) extended Liu’s framework to the case of stock model with mean-reverting process, and Yao (2012) proved the no-arbitrage determinant theorems for this type of model. Chen, Liu and Ralescu (2013) considered the case of stock with dividends and proposed an uncertain stock model with periodic dividends and derived the pricing formulas of some options under this model. Besides, Chen and Gao (2013) investigated the interest term structure within the framework of uncertainty theory and obtained the zero-coupon bond price formula for uncertain interest rate model. Research on currency option within the framework of uncertainty theory began with Liu, Chen and Ralescu (2015) in which uncertain differential equations were employed to model currency exchange rate and some currency option price formulas were derived.

This paper is concerned with solving the problem of valuation of stock loan within the framework of uncertainty theory. Considering the stock prices fluctuate around some average level in long run, we will investigate the valuation of stock loan under uncertain mean-reverting stock model. The rest of the paper is organized as follows. In next section, some useful concepts and theorems of uncertainty theory as needed are stated. In Section 3, the valuation of stock loan for uncertain mean-reverting stock model is investigated. In Section 4, we explore the pricing problem of capped stock loan for this type of stock model. Finally, a brief conclusion is given in Section 5.

2 Preliminary

The following is some useful definitions and theorems of uncertainty theory as needed.

**Definition 2.1** (Liu 2007) Let \( \Gamma \) be a nonempty set, and let \( \mathcal{L} \) be a \( \sigma \)-algebra over \( \Gamma \). An uncertain measure is a function \( \mathcal{M} : \mathcal{L} \rightarrow [0, 1] \) such that

**Axiom 1.** (Normality Axiom) \( \mathcal{M}\{\Gamma\} = 1 \) for the universal set \( \Gamma \);

**Axiom 2.** (Duality Axiom) \( \mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1 \) for any event \( \Lambda \);
Axiom 3. (Subadditivity Axiom) For every countable sequence of events \( \{ \Lambda_i \} \) we have
\[
M \left\{ \bigcup_{i=1}^{\infty} \Lambda_i \right\} \leq \sum_{i=1}^{\infty} M\{ \Lambda_i \}. \tag{2.1}
\]

A set \( \Lambda \in \mathcal{L} \) is called an event. The uncertain measure \( M\{ \Lambda \} \) indicates the degree of belief that \( \Lambda \) will occur. The triplet \((\Gamma, \mathcal{L}, M)\) is called an uncertainty space. In order to obtain an uncertain measure of compound event, a product uncertain measure was defined by Liu (2009).

Axiom 4. (Product Axiom) Let \((\Gamma_k, \mathcal{L}_k, M_k)\) be uncertainty spaces for \( k = 1, 2, \cdots \). The product uncertain measure \( M \) is an uncertain measure on the product \( \sigma \)-algebra \( \mathcal{L}_1 \times \mathcal{L}_2 \times \cdots \) satisfying
\[
M \left\{ \prod_{k=1}^{\infty} \Lambda_k \right\} = \bigwedge_{k=1}^{\infty} M_k \{ \Lambda_k \} \tag{2.2}
\]
where \( \Lambda_k \) are arbitrarily chosen events from \( \mathcal{L}_k \) for \( k = 1, 2, \cdots \), respectively.

Definition 2.2 (Liu 2007) An uncertain variable is a measurable function from an uncertainty space \((\Gamma, \mathcal{L}, M)\) to the set of real numbers, i.e., \( \{ \xi \in B \} \) is an event for any Borel set \( B \).

Definition 2.3 (Liu 2007) The uncertainty distribution \( \Phi \) of an uncertain variable \( \xi \) is defined by
\[
\Phi(x) = M\{ \xi \leq x \} \tag{2.3}
\]
for any real number \( x \).

Definition 2.4 (Liu 2007) An uncertain variable \( \xi \) is called normal if it has a normal uncertainty distribution
\[
\Phi(x) = \left( 1 + \exp \left( \frac{\pi(e - x)}{\sqrt{3}\sigma} \right) \right)^{-1} \tag{2.4}
\]
denoted by \( \mathcal{N}(e, \sigma) \) where \( e \) and \( \sigma \) are real numbers with \( \sigma > 0 \).

Definition 2.5 (Liu 2010) An uncertainty distribution \( \Phi(x) \) is said to be regular if it is a continuous and strictly increasing function with respect to \( x \) at which \( 0 < \Phi(x) < 1 \), and
\[
\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1. \tag{2.5}
\]

Definition 2.6 (Liu 2010) Let \( \xi \) be an uncertain variable with regular uncertainty distribution \( \Phi(x) \). Then the inverse function \( \Phi^{-1}(\alpha) \) is called the inverse uncertainty distribution of \( \xi \).

Definition 2.7 (Liu 2007) Let \( \xi \) be an uncertain variable. Then the expected value of \( \xi \) is defined by
\[
E[\xi] = \int_{0}^{+\infty} M\{ \xi \geq r \}dr - \int_{-\infty}^{0} M\{ \xi \leq r \}dr \tag{2.6}
\]
provided that at least one of the two integrals is finite.

**Theorem 2.1** (Liu 2007) Let $\xi$ be an uncertain variable with uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^0 \Phi(x)dx.$$  \hfill (2.7)

**Theorem 2.2** (Liu 2010) Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. Then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\,d\alpha.$$ \hfill (2.8)

**Theorem 2.3** (Liu 2010) Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If the function $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then the uncertain variable

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$$ \hfill (2.9)

has an inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \cdots, \Phi_n^{-1}(1 - \alpha)).$$ \hfill (2.10)

Liu and Ha (2010) proved that the uncertain variable $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ has an expected value

$$E[\xi] = \int_0^1 f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \cdots, \Phi_n^{-1}(1 - \alpha))\,d\alpha.$$ \hfill (2.11)

An uncertain process is a sequence of uncertain variables indexed by a totally ordered set $T$. A formal definition is given below.

**Definition 2.8** (Liu 2008) Let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space and let $T$ be a totally ordered set (e.g. time). An uncertain process is a function $X_t(\gamma)$ from $T \times (\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{X_t \in B\}$ is an event for any Borel set $B$ at each time $t$.

**Definition 2.9** (Liu 2009) An uncertain process $C_t$ is said to be a canonical Liu process if

(i) $C_0 = 0$ and almost all sample paths are Lipschitz continuous,

(ii) $C_t$ has stationary and independent increments,

(iii) every increment $C_{s+t} - C_s$ is a normal uncertain variable with expected value 0 and variance $t^2$.
In order to deal with the integration and differentiation of uncertain processes, Liu (2009) proposed an uncertain integral with respect to canonical Liu process.

**Definition 2.10** (Liu 2009) Let $X_t$ be an uncertain process and $C_t$ be a canonical Liu process. For any partition of closed interval $[a, b]$ with $a = t_1 < t_2 < \cdots < t_{k+1} = b$, the mesh is defined as

$$
\Delta = \max_{1 \leq i \leq k} |t_{i+1} - t_i|.
$$

Then the Liu integral of $X_t$ is defined as

$$
\int_a^b X_t dC_t = \lim_{\Delta \to 0} \sum_{i=1}^{k} X_{t_i} (C_{t_{i+1}} - C_{t_i})
$$

provided that the limit exists almost surely and is finite. In this case, the uncertain process $X_t$ is said to be Liu integrable.

**Definition 2.11** (Chen-Ralescu 2013) Let $C_t$ be a canonical Liu process and let $Z_t$ be an uncertain process. If there exist uncertain processes $\mu_t$ and $\sigma_t$ such that

$$
Z_t = Z_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dC_s
$$

for any $t \geq 0$, then $Z_t$ is called a Liu process with drift $\mu_t$ and diffusion $\sigma_t$. Furthermore, $Z_t$ has an uncertain differential

$$
dZ_t = \mu_t dt + \sigma_t dC_t.
$$

Liu (2009) verified the fundamental theorem of uncertain calculus, i.e., for a canonical Liu process $C_t$ and a continuous differentiable function $h(t, c)$, the uncertain process $Z_t = h(t, C_t)$ is differentiable and has a Liu differential

$$
dZ_t = \frac{\partial h}{\partial t}(t, C_t) dt + \frac{\partial h}{\partial c}(t, C_t) dC_t.
$$

**Definition 2.12** (Yao-Chen 2013) Let $\alpha$ be a number with $0 < \alpha < 1$. An uncertain differential equation

$$
dX_t = f(t, X_t) dt + g(t, X_t) dC_t
$$

is said to have an $\alpha$-path $X_t^\alpha$ if it solves the corresponding ordinary differential equation

$$
dX_t^\alpha = f(t, X_t^\alpha) dt + |g(t, X_t^\alpha)| \Phi^{-1}(\alpha) dt
$$
where $\Phi^{-1}(\alpha)$ is the inverse standard normal uncertainty distribution, i.e.,

$$\Phi^{-1}(\alpha) = \frac{\sqrt{3}}{\pi} \ln \frac{\alpha}{1 - \alpha}. \quad (2.18)$$

**Theorem 2.4** (Yao-Chen Formula 2013) Let $X_t$ and $X_t^\alpha$ be the solution and $\alpha$-path of the uncertain differential equation

$$dX_t = f(t, X_t)dt + g(t, X_t) dC_t, \quad (2.19)$$

respectively. Then

$$M \{X_t \leq X_t^\alpha, \forall t\} = \alpha, \quad (2.20)$$

$$M \{X_t > X_t^\alpha, \forall t\} = 1 - \alpha. \quad (2.21)$$

**Theorem 2.5** (Yao-Chen 2013) Let $X_t$ and $X_t^\alpha$ be the solution and $\alpha$-path of the uncertain differential equation

$$dX_t = f(t, X_t)dt + g(t, X_t) dC_t, \quad (2.22)$$

respectively. Then the solution $X_t$ has an inverse uncertainty distribution

$$\Psi_t^{-1}(\alpha) = X_t^\alpha. \quad (2.23)$$

**Theorem 2.6** (Yao 2013) Let $X_t$ and $X_t^\alpha$ be the solution and $\alpha$-path of the uncertain differential equation

$$dX_t = f(t, X_t)dt + g(t, X_t) dC_t, \quad (2.24)$$

respectively. Then for any time $s > 0$ and strictly increasing function $J(x)$, the supremum

$$\sup_{0 \leq t \leq s} J(X_t) \quad (2.25)$$

has an inverse uncertainty distribution

$$\Psi_s^{-1}(\alpha) = \sup_{0 \leq t \leq s} J(X_t^\alpha); \quad (2.26)$$

and the infimum

$$\inf_{0 \leq t \leq s} J(X_t) \quad (2.27)$$

has an inverse uncertainty distribution

$$\Psi_s^{-1}(\alpha) = \inf_{0 \leq t \leq s} J(X_t^\alpha). \quad (2.28)$$
3 Valuation of stock loan

Liu (2009) suggested to describe the stock price process by using an uncertain differential equation and proposed an uncertain stock model as follows

\[
\begin{align*}
\frac{dX_t}{X_t} &= r dt \\
\frac{dS_t}{S_t} &= \mu S_t dt + \sigma S_t dC_t
\end{align*}
\]

(3.1)

where \(X_t\) is the bond price, \(S_t\) is the stock price, \(r\) is the riskless interest rate, \(\mu\) is the log-drift, \(\sigma\) is the log-diffusion, and \(C_t\) is a canonical Liu process.

Considering the case of the stock price usually fluctuates around some average price in long run, Peng and Yao (2011) extended Liu's uncertain stock model to an uncertain mean-reverting stock model as follows

\[
\begin{align*}
\frac{dX_t}{X_t} &= r dt \\
\frac{dS_t}{S_t} &= (m - a S_t) dt + \sigma dC_t
\end{align*}
\]

(3.2)

where \(X_t\) is the bond price, \(S_t\) is the stock price, \(r\) is the riskless interest rate, \(m\), \(a\) and \(\sigma\) are constants, and \(C_t\) is a canonical Liu process. This type of model can be used to capture price movements that have the tendency to move towards an equilibrium level.

Suppose a borrower can obtain amount \(K\) from a bank with one share of his or her stock as collateral. After paying a service fee \(c\) (\(0 < c < K\)) to the bank, the borrower receives the amount \((K - c)\). The borrower has the right to redeem the stock at any time prior to the loan maturity \(T\) by repaying the bank the principal plus interest associated to the loan that is \(K \exp(\theta t)\), where \(\theta > r\) is the loan interest rate. A basic problem on stock loan is what are the fair value of the principal \(K\), the loan interest \(\theta\) and the fee \(c\) charged by the bank for providing the service. The main objective of this paper is to evaluate the stock loan value, in turn it can be used to determine the rational values of the parameters \(K\), \(\theta\) and \(c\).

From the above description on stock loan, we can see that the stock loan means that the borrower pays \(S_0 - (K - c)\) to buy an American option with a time-dependent strike price \(K \exp(\theta t)\) and maturity \(T\) at time 0. The present value of the payoff of the borrower is

\[
\sup_{0 \leq t \leq T} \exp(-rt)[S_t - K \exp(\theta t)]^+.
\]

(3.3)

Thus the value of the stock loan should be the expected present value of the payoff.
**Definition 3.1** Assume a stock loan has loan amount $K$, loan interest rate $\theta$ and loan maturity time $T$. Then the value of the stock loan is

$$f = E \left[ \sup_{0 \leq t \leq T} \exp(-rt)[S_t - K \exp(\theta t)]^+ \right].$$

(3.4)

**Theorem 3.1** Assume a stock loan for the stock model (3.2) has loan amount $K$, loan interest rate $\theta$ and loan maturity time $T$. Then the value of the stock loan is

$$f = \int_0^1 \sup_{0 \leq t \leq T} \exp(-rt)[S_t^\alpha - K \exp(\theta t)]^+ d\alpha$$

(3.5)

where $S_t^\alpha = \frac{1}{a} (m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0$.

**proof:** Solving the uncertain differential equation

$$dS_t^\alpha = (m - aS_t^\alpha)dt + \sigma \Phi^{-1}(\alpha)dt$$

(3.6)

where $0 < \alpha < 1$ and $\Phi^{-1}(\alpha)$ is the inverse standard normal uncertainty distribution, we have

$$S_t^\alpha = \frac{1}{a} (m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0.$$  

(3.7)

That means that the uncertain differential equation

$$dS_t = (m - aS_t)dt + \sigma dC_t$$

(3.8)

has an $\alpha$-path

$$S_t^\alpha = \frac{1}{a} (m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0.$$  

(3.9)

Since $J(x) = \exp(-rt)[x - K \exp(\theta t)]^+$ is an increasing function, it follows from Theorem 2.6 that

$$\sup_{0 \leq t \leq T} J(S_t) = \sup_{0 \leq t \leq T} \exp(-rt)[S_t - K \exp(\theta t)]^+$$

has an inverse uncertainty distribution

$$\sup_{0 \leq t \leq T} \exp(-rt)[S_t^\alpha - K \exp(\theta t)]^+.$$  

(3.10)

Therefore the value of the stock loan is

$$f = \int_0^1 \sup_{0 \leq t \leq T} \exp(-rt)[S_t^\alpha - K \exp(\theta t)]^+ d\alpha$$

(3.11)

where $S_t^\alpha = \frac{1}{a} (m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0$. 


4 Valuation of capped stock loan

In this section, we study the valuation of capped stock loan, in which there is a capped limit for the stock price. Setting up such a capped limit for stock price, the bank can avoid the possible large loss in the future time. Thus capped stock loans has more advantages than standard loans in which the borrower still has the possibility of obtaining a profit, and the bank may cut down future risk in the meantime.

In capped stock loan, after paying back the loan, the borrower will get the minimum value between the predetermined money and the stock price. There are two types of cap: one is constant cap, another is the cap with a constant growth rate.

Suppose a stock loan has loan amount $K$, loan interest rate $\theta$, and loan maturity time $T$. Assume the loan has constant cap $L$. Then the present value of the payoff of the borrower is

$$
\sup_{0 \leq t \leq T} \exp(-rt)[S_t \land L - K \exp(\theta t)]^{-}.
$$

(4.1)

Thus the value of the capped stock loan should be the expected present value of the payoff.

**Definition 4.1** Assume a stock loan with constant cap $L$ has loan amount $K$, loan interest rate $\theta$ and loan maturity time $T$. Then the value of the capped stock loan is

$$
f = E \left[ \sup_{0 \leq t \leq T} \exp(-rt)[S_t \land L - K \exp(\theta t)]^{-} \right].
$$

(4.2)

**Theorem 4.1** Assume a stock loan for the stock model (3.2) has loan amount $K$, loan interest rate $\theta$, constant cap $L$ and loan maturity time $T$. Then the value of the capped stock loan is

$$
f = \int_{0}^{1} \sup_{0 \leq t \leq T} \exp(-rt) [S_t^{-} \land L - K \exp(\theta t)]^{-} \, d\alpha
$$

(4.3)

where $S_t^{-} = \frac{1}{a}(m + \sigma\Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0$.

**proof:** Since $J(x) = \exp(-rt)[x \land L - K \exp(\theta t)]^{-}$ is an increasing function, it follows from Theorem 2.6 that $\sup_{0 \leq t \leq T} J(S_t) = \sup_{0 \leq t \leq T} \exp(-rt)[S_t \land L - K \exp(\theta t)]^{-}$ has an inverse uncertainty distribution

$$
\sup_{0 \leq t \leq T} \exp(-rt) [S_t^{-} \land L - K \exp(\theta t)]^{-}.
$$

(4.4)

Therefore the value of the stock loan is

$$
f = \int_{0}^{1} \sup_{0 \leq t \leq T} \exp(-rt) [S_t^{-} \land L - K \exp(\theta t)]^{-} \, d\alpha
$$

(4.5)
where $S_t^\alpha = \frac{1}{a}(m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0$.

The cap with a constant growth rate is a time-varying cap that grows at a constant rate $\beta > 0$ which actually is a function of time, that is

$$L_t = L \exp(\beta t).$$

(4.6)

Suppose a stock loan with cap $L_t$ given by (4.6) has loan amount $K$, loan interest rate $\theta$, and loan maturity time $T$. Then the present value of the payoff of the borrower is

$$\sup_{0 \leq t \leq T} \exp(-rt)[S_t \land L \exp(\beta t) - K \exp(\theta t)]^+.\quad (4.7)$$

Thus the value of the stock loan should be the expected present value of the payoff.

**Definition 4.2** Assume a stock loan with time-varying cap $L \exp(\beta t)$ has loan amount $K$, loan interest rate $\theta$, and loan maturity time $T$. Then the value of the stock loan is

$$f = E \left[ \sup_{0 \leq t \leq T} \exp(-rt)[S_t \land L \exp(\beta t) - K \exp(\theta t)]^+ \right].\quad (4.8)$$

**Theorem 4.2** Assume a stock loan for the stock model (3.2) has loan amount $K$, loan interest rate $\theta$, constant growth rate cap $L \exp(\beta t)$ and loan maturity time $T$. Then the value of the stock loan is

$$f = \int_0^1 \sup_{0 \leq t \leq T} \exp(-rt)[S_t^\alpha \land L \exp(\beta t) - K \exp(\theta t)]^+ d\alpha\quad (4.9)$$

where $S_t^\alpha = \frac{1}{a}(m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0$.

**proof:** Since $J(x) = \exp(-rt)[x \land L \exp(\beta t) - K \exp(\theta t)]^+$ is an increasing function, it follows from Theorem 2.6 that $\sup_{0 \leq t \leq T} J(S_t) = \sup_{0 \leq t \leq T} \exp(-rt)[S_t \land L \exp(\beta t) - K \exp(\theta t)]^+$ has an inverse uncertainty distribution

$$\sup_{0 \leq t \leq T} \exp(-rt)[S_t^\alpha \land L \exp(\beta t) - K \exp(\theta t)]^+.\quad (4.10)$$

Therefore the value of the stock loan is

$$f = \int_0^1 \sup_{0 \leq t \leq T} \exp(-rt)[S_t^\alpha \land L \exp(\beta t) - K \exp(\theta t)]^+ d\alpha\quad (4.11)$$

where $S_t^\alpha = \frac{1}{a}(m + \sigma \Phi^{-1}(\alpha))(1 - \exp(-at)) + \exp(-at)S_0$.

5 Conclusion

In this paper, within the framework of uncertainty theory, under the assumption that the underlying stock price following a mean reverting uncertain differential equation driven by canonical Liu process,
the valuation of stock loan was investigated, and the price formulas of standard stock loan and capped stock loan were derived with the method of uncertain calculus.

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