Abstract

A multi-product newsboy problem with uncertain demand and uncertain storage space is investigated under a warehousing chance constraint. It is assumed that indeterminacy may appear in demand and storage space due to rough estimation of their distributions by decision-maker. Uncertainty theory provides a new tool to deal with the human uncertainty. We develop an uncertain expected value programming model (UEVPM) based on uncertainty theory. Uncertain variables (functions from uncertainty space to the set of real numbers) are used to describe subjective estimation. The objective function is to maximize the expected profit of newsboy at a predetermined confidence level. Furthermore, we convert the chance constrained uncertain programming model to its equivalent deterministic form so as to be solved by classic integer programming method. Finally, three numerical examples are given to illustrate the potential usefulness of the study.

Keywords: Newsboy problem; Chance constraint; Multi-product; Uncertain programming; Uncertainty theory

1. Introduction

The classic newsboy problem deals with situations where the demand for products is random and those products that are ordered but remain unsold at the end of the cycle become obsolete. Many researchers have done lots of work on stochastic multi-product newsboy problem under different constraints. Hadley and Whitin [1] presented a multi-product problem with a single budget constraint. After Hadley and Whitin’s seminal work,
many researches on multi-product newsboy problem have been developed. Nahmias and Schmidt [2] proposed a multiproduct newsboy problem with stochastic demands subject to linear and deterministic constraint on space or budget. Lau and Lau [3] extended the newsboy problem to handle general demand distributions and provided an efficient solution for a multi-product multi-constraint newsboy problem. In addition, Abdel-Malek et al. [4] presented a multiproduct newsboy model under a budget constraint with probabilistic demand and random yield. Following previous work, Ozler et al. [5] proposed the multi-product newsboy problem under a Value at Risk (VaR) constraint. Another extension is attributed to Vairaktarakis [6] where he considered newsboy models with a budget constraint, and assumed demand can be described by using discrete or interval scenarios.

However, in our daily life, we are often lack of observed data about the events, not only for economic reasons or technical difficulties, but also for influence of unexpected event. Because there are not enough data to get probability distribution of events, we have to invite some domain experts to give belief degree that each event would happen. Human beings usually overweight unlikely events (Kahneman and Tversky [7]). Thus the belief degree may have much larger variance than the real frequency. In this situation, if we insist on dealing with the belief degree using probability theory, some counterintuitive results will be obtained (Liu [8]).

In order to deal with belief degree, an uncertainty theory was founded by Liu [9] in 2007 and refined by Liu [10] in 2010. The uncertainty theory has been a branch of mathematics to deal with human uncertainty. Many researchers have done a lot of significant work in this area. Liu [11] proved the linearity of expected value operator. Liu and Ha [12] derived a formula that can easily calculate the expected values of monotone functions of uncertain variables. As an important contribution, Liu [13] first proposed uncertain programming that is a type of mathematical programming involving uncertain variables. Many practical problems can be expressed as uncertain programming problems. A number of works in this area have also been developed. Liu and Chen [14] proposed an uncertain multi-objective programming and an uncertain goal programming. Liu and Yao [15] successfully
presented an uncertain multilevel programming. Recently, uncertain programming has been extended to the fields of uncertain network optimization [16, 17], economic order quantity model [18], Chinese postman problem [19], transportation problem [20], project scheduling problem [21], reverse logistics network design problem [22], Euler index of uncertain graph [23], aggregate production planning model [24], etc.

The estimation of market demand is always the key factor that affects the total profit. In order to deal with the indeterminacy information, some researchers applied uncertainty theory to newsboy problem. Qin and Kar [25] assumed that the market demand is an uncertain variable and introduced uncertainty theory into newsboy problem. They found the optimal order quantity to maximize the expected profit. Gao [26] took setup cost and initial stock into account and derived the optimal (s, S) policy for the uncertain newsboy problem. However, the above two research works considered only one product.

This paper extends uncertain single-product newsboy problem to uncertain multi-product newsboy problem whose demand and storage space of each product are not deterministic but estimated by experts. Decision-makers want to take decisions with the maximum expected profit subject to chance constraint, then expected value model is formulated for the multi-product newsboy problem. Under the framework of uncertainty theory, it can be transformed into the equivalent model and solved by classic integer programming method.

The remainder of this paper is organized as follows. In Section 2, some basic concepts and properties of uncertainty theory used throughout this paper are introduced. In Section 3, we present an uncertain expected value model under a warehousing chance constraint and convert the model to an equivalent deterministic integer programming model. Three numerical examples are given in Section 4. Section 5 gives a brief summary to this paper.

2. Preliminaries

In this section, we introduce some foundational concepts and properties of uncertainty theory, which are used throughout this paper.

Let $\Gamma$ be a nonempty set, and let $\mathcal{L}$ be a $\sigma$-algebra over $\Gamma$. Each element $\Lambda \in \mathcal{L}$
is assigned a number $\mathcal{M}\{\Lambda\} \in [0,1]$. In order to ensure that the $\mathcal{M}\{\Lambda\}$ has certain mathematical properties, Liu [9, 10] presented four axioms: (1) normality, (2) duality, (3) subadditivity, and (4) product axiom.

**Definition 1.** (Liu [9]) Let $\Gamma$ be a nonempty set, $\mathcal{L}$ a $\sigma$-algebra over $\Gamma$, and $\mathcal{M}$ an uncertain measure. Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

**Definition 2.** (Liu [9]) An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set $B$ of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

**Definition 3.** (Liu [11]) The uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n} \{\xi_i \in B_i\}\right\} = \min_{1 \leq i \leq n} \mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \cdots, B_n$ of real numbers.

The uncertainty distribution of an uncertain variable $\xi$ is defined by $\Phi(x) = \mathcal{M}\{\xi \leq x\}$ for any real number $x$. For example, the linear uncertain variable $\xi \sim \mathcal{L}(a, b)$ has an uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x-a)/(b-a), & \text{if } a \leq x \leq b \\ 1, & \text{if } b \leq x. \end{cases}$$

**Definition 4.** (Liu [10]) An uncertainty distribution $\Phi$ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$.

Obviously, linear uncertain variable has a regular uncertainty distribution.

**Definition 5.** (Liu [10]) Let $\xi$ be an uncertain variable with regular uncertainty distribution $\Phi$. If the expected value exists, then

$$E[\xi] = \int_{0}^{1} \Phi^{-1}(\alpha)d\alpha.$$
Theorem 1. (Liu [10]) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f$ is a strictly increasing function, then

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$$

is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)).$$

Assuming that demand and storage space are uncertain variables, we will investigate an expected value model for multi-product newsboy problem in Section 3.

3. Expected Profit Model and Its Deterministic Equivalent Class

In order to introduce the uncertain multi-product newsboy problem clearly, we list model parameters and decision variables as follows:

$n$: number of products;

$\xi_i$: quantity demanded for product $i$, an uncertain variable;

$\eta_i$: storage space needed per unit of product $i$, an uncertain variable;

$x_i$: order quantity for product $i$, a decision variable;

$p_i$: unit selling price for product $i$;

$q_i$: unit procurement cost for product $i$;

$h_i$: unit salvage value for product $i$;

$V$: total storage capacity;

$\beta$: predetermined confidence level;

$f_i(x_i, \xi_i)$: the profit function for the order quantity $x_i$ and demand $\xi_i$;

$F(\mathbf{x}, \mathbf{\xi})$: the total profit function for all products, in which $\mathbf{x}=(x_1, x_2, \cdots, x_n)$, $\mathbf{\xi} = (\xi_1, \xi_2, \cdots, \xi_n)$.

In classical newsboy problem, the demands $\xi_i, i = 1, 2, \ldots, n$, and storage spaces $\eta_i, i = 1, 2, \cdots, n$, of products are regarded as random variables, and they are obtained by statistics methods. Unfortunately, sometimes we cannot get enough historical data, or historical data are invalid because of the changing conditions. In this situation, the
demand and storage space data can only be obtained from the decision-makers’ empirical estimation in a practical way. Therefore, it is not suitable to regard these empirical estimation data as random variables. In this paper, we employ uncertain variables to describe the demand and storage space of each product.

Obviously, we have \( p_i > q_i > h_i > 0 \). Then the profits for products \( i \) are given by

\[
\begin{align*}
    f_i(x_i, \xi_i) = \\
    \begin{cases}
        (p_i - q_i)x_i, & \text{if } x_i \leq \xi_i \\
        p_i\xi_i + (x_i - \xi_i)h_i - q_i x_i, & \text{if } x_i > \xi_i
    \end{cases}
\end{align*}
\]

for \( i = 1, 2, \cdots, n \), respectively, and the total profit of the newsboy is as follow,

\[
    F(x, \xi) = \sum_{i=1}^{n} f_i(x_i, \xi_i).
\]

In real life, as \( \xi_i, i = 1, 2, \cdots, n \), go up, we will sell more products and make much more profit. In practical application, we usually assume that uncertainty distributions of the demands are regular. Hence, these uncertainty distributions are continuous and strictly increasing at each point \( x_i \) with \( 0 < \Phi_i(x_i) < 1, i = 1, 2, \cdots, n \). Otherwise, a small perturbation can be imposed to obtain a regular one. Therefore, the profits \( f_i(x_i, \xi_i) \) with respect to \( \xi_i, i = 1, 2, \cdots, n \), are strictly increasing functions.

Since the demands are uncertain variables, \( f_i(x_i, \xi_i), i = 1, 2, \cdots, n \), are also uncertain variables. In order to rank uncertain variables, we give the following definition based on the uncertainty theory.

**Definition 6.** If \( E[F(x^*, \xi)] \geq E[F(x, \xi)] \) for all feasible order quantity \( x \), then we call \( E[F(x^*, \xi)] \) as the maximum expected profit of the newsboy.

In this part, we formulate an uncertain multi-product newsboy problem to find optimal order quantity and maximize the expected value of the profit \( F(x, \xi) \) with uncertain storage space constraint. The expected value model can be expressed as
\[
\begin{aligned}
\max_{x} E[F(x, \xi)] \\
\text{subject to :} \\
M \left\{ \sum_{i=1}^{n} x_i \eta_i \leq V \right\} \geq \beta, \ i = 1, 2, \cdots, n \quad (A2) \\
x_i \geq 0, \ i = 1, 2, \cdots, n, \text{ integers.} \quad (A3)
\end{aligned}
\]

The objective function (A1) is the expected value of profit. Constraints (A2) mean that the total storage space of products ordered by the newsboy is less than \(V\) with confidence level \(\beta\). Constraints (A3) assure decision variables \(x_i, i = 1, 2, \cdots, n\), are nonnegative integers.

In order to solve this model, we analyze the properties and give the following theorem.

**Theorem 2.** Let \(x_1, x_2, \cdots, x_n\) be nonnegative decision variables, \(\xi_i, \eta_i\) be nonnegative independent uncertain variables with regular uncertainty distributions \(\Phi_i, \Upsilon_i, i = 1, 2, \cdots, n\) respectively. And let \(x=(x_1, x_2, \cdots, x_n)\). Then the model (1) is equivalent to the following deterministic programming model,

\[
\begin{aligned}
\max_{x} H(x) \\
\text{subject to :} \\
\sum_{i=1}^{n} x_i \Upsilon_i^{-1}(\beta) \leq V, \ i = 1, 2, \cdots, n \quad (B2) \\
x_i \geq 0, \ i = 1, 2, \cdots, n, \text{ integers} \quad (B3)
\end{aligned}
\]

where

\[
H(x) = \sum_{i=1}^{n} \left[ (p_i - q_i)x_i - (p_i - h_i) \int_{0}^{\Phi_i(x_i)} (x_i - \Phi_i^{-1}(\alpha))d\alpha \right].
\]

**Proof:** Note that \(f_i(x_i, \xi_i)\) are strictly increasing functions with respect to \(\xi_i, i = 1, 2, \cdots, n\). Thus the inverse uncertainty distributions of \(f_i(x_i, \xi_i)\) are

\[
\psi_i^{-1}(\alpha) = \begin{cases} 
(p_i - q_i)x_i, & \text{if } \alpha \geq \Phi_i(x_i) \\
(h_i - q_i)x_i + (p_i - h_i)\Phi_i^{-1}(\alpha), & \text{if } \Phi_i(x_i) > \alpha
\end{cases}
\]

for \(i = 1, 2, \cdots, n\), respectively.
Since $\xi_i, i = 1, 2, \cdots, n$ are assumed to be uncertain variables, $f_i(x_i, \xi_i)$, as functions of $\xi_i, i = 1, 2, \cdots, n$, are also uncertain variables. According to Definition 5, the expected value of $f_i(x_i, \xi_i)$ is

$$E[f_i(x_i, \xi_i)] = \int_0^1 \psi_i^{-1}(\alpha) d\alpha$$

$$=\int_0^{\Phi_i(x_i)} (p_i - q_i)x_i d\alpha + \int_0^{\Phi_i(x_i)} (h_i - q_i)x_i + (p_i - h_i)\Phi_i^{-1}(\alpha) d\alpha$$

$$= (p_i - q_i)x_i(1 - \Phi_i(x_i)) + (h_i - q_i)x_i\Phi_i(x_i) + (p_i - h_i)\int_0^{\Phi_i(x_i)} \Phi_i^{-1}(\alpha) d\alpha$$

$$= (p_i - q_i)x_i - (p_i - h_i)\int_0^{\Phi_i(x_i)} (x_i - \Phi_i^{-1}(\alpha)) d\alpha.$$

Since $F(x, \xi) = \sum_{i=1}^n f_i(x_i, \xi_i)$ is a linear function of uncertain variables $f_i(x_i, \xi_i), i = 1, 2, \cdots, n$, it follows from Theorem 1 that the expected value of $F(x, \xi)$ is

$$H(x) = \sum_{i=1}^n E[f_i(x_i, \xi_i)] = \sum_{i=1}^n \left[ (p_i - q_i)x_i - (p_i - h_i)\int_0^{\Phi_i(x_i)} (x_i - \Phi_i^{-1}(\alpha)) d\alpha \right].$$

Thus, the objective function of model (1) is equivalent to maximizing $H(x)$ for each feasible solution which satisfies the constraint conditions.

Next, we prove the constraints (A2) are equivalent to the deterministic inequality constraints (B2). Let $\eta= f_i(x_1, x_2, \cdots, x_n, \eta_1, \eta_2, \cdots, \eta_n) = \sum_{i=1}^n x_i\eta_i$. Please note that $\eta$ is a strictly increasing function with respect to $\eta_i, i = 1, 2, \cdots, n$. Denote the uncertainty distribution of $\eta$ as $\Delta$. According to Theorem 1,

$$\Delta^{-1}(\beta) = \sum_{i=1}^n x_i\Upsilon_i^{-1}(\beta).$$

This means

$$M \left\{ \eta \leq \sum_{i=1}^n x_i\Upsilon_i^{-1}(\beta) \right\} = M \{ \eta \leq \Delta^{-1}(\beta) \} = \beta.$$

In addition, since uncertainty distribution of $\eta$ is a strictly increasing function, from

$$M \{ \eta \leq V \} \geq \beta,$$
we know
\[ \sum_{i=1}^{n} x_i \Upsilon_i^{-1}(\beta) \leq V. \]

The proof is completed.

In fact, the model (2) becomes an integer programming problem in deterministic environment which can be solved by the classical integer programming algorithm. Branch and bound algorithm is presented as below.

**Step 1:** Set \( Z^* = 0 \).

**Step 2:** Put the original problem on the candidate list.

**Step 3:** Select a problem \( S \) from the candidate list, relax its integer constraints with the continuous ones and yield a relaxation. Use penalty function to handle constraint. Solve the relaxation by quasi-newton method to obtain the bound \( u(S) \).

(a) If the relaxation is infeasible, node can be pruned.

(b) Otherwise, if \( u(S) \leq Z^* \), node can be pruned.

(c) Otherwise, if \( u(S) > Z^* \), and the solution is feasible for the integer programming, set \( Z^* = u(S) \).

(d) Otherwise, pick one of the fractional variables, say \( x_i \) with its current value \( x_i^* \), and create two subproblems by adding the respective constraints: \( x_i \leq \lfloor x_i^* \rfloor \) and \( \lfloor x_i^* \rfloor + 1 \leq x_i \). Then, add the new subproblems to the candidate list, use penalty function to handle constraint and apply quasi-newton method to each subproblem to possibly obtain an improved feasible solution and a corresponding better bound.

**Step 4:** If the candidate list is nonempty, go to Step 2. Otherwise, the algorithm is completed.

Analysis of the feasible regions of decision variables makes it possible to improve speed of the algorithm. Because the \( i \)th product has its storage space requirement and the total available storage space cannot be changed, we can calculate the maximum quantity of the
ith product that satisfies the storage space constraint. For example, the ith product has an uncertainty distribution \( L(30, 40) \), and the total storage capacity is 1000 \( cm^3 \). This indicates if we order the ith product, it will occupy at least 30 \( cm^3 \) storage space, and the maximum order quantity of the ith product is \( \lfloor 1000/30 \rfloor = 33 \) (\( \lfloor x \rfloor \) is the greatest integer that is equal to or lesser than \( x \)). Hence, the maximum order quantity of each product is equal or lesser than \( \lfloor V/a_i \rfloor \) units. In addition, obviously, if we order nothing, \( x = (0, 0, 0) \) is a feasible solution and a lower bound of objective function is \( Z^* = 0 \). In next section, we will employ the branch and bound algorithm to solve several examples.

4. Numerical examples

In this section, we consider three examples to illustrate the modeling idea. Suppose that a merchant will order Christmas toys before the beginning of the season. There are different kinds of Christmas toys in his shopping list. In order to make more profit, he evaluates the demand and storage space distribution of each Christmas toys based on his experience. We assume that the demands and storage space of products are linear uncertain variables.

**Example 1.** Suppose that there are two kinds of Christmas toys and confidence level \( \beta = 0.8 \). Other parameters of the model are listed in Table 1. As shown in Table 1, the two products have the same uncertainty distributions of demand and storage space, and also have the same salvage value. Moreover, the gap between selling price and purchasing price of the two products is also equal 10.

<table>
<thead>
<tr>
<th>Product</th>
<th>( p_i ) ($/unit)</th>
<th>( q_i ) ($/unit)</th>
<th>( h_i ) ($/unit)</th>
<th>( \xi_i )</th>
<th>( \eta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>10</td>
<td>6</td>
<td>( L(0, 200) )</td>
<td>( L(30, 40) )</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>5</td>
<td>4</td>
<td>( L(0, 200) )</td>
<td>( L(30, 40) )</td>
</tr>
</tbody>
</table>

The optimal solutions are shown in Table 2. Obviously, when storage space con-
straint changes from 5000 \( cm^3 \) to 12000 \( cm^3 \), the expected profit increases from $1045.7 to $1621.9. It is worth mentioning that order quantity of Product 1 is always less than that of Product 2.

Table 2: Optimal order quantity and expected profit with different storage space constraints—Example 1.

<table>
<thead>
<tr>
<th>Storage space ((cm^3))</th>
<th>Expected profit ( ($) )</th>
<th>Order quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>1045.7</td>
<td>58  73</td>
</tr>
<tr>
<td>6000</td>
<td>1190.4</td>
<td>69  88</td>
</tr>
<tr>
<td>7000</td>
<td>1318.6</td>
<td>81  103</td>
</tr>
<tr>
<td>8000</td>
<td>1420.8</td>
<td>92  118</td>
</tr>
<tr>
<td>9000</td>
<td>1502.3</td>
<td>104 132</td>
</tr>
<tr>
<td>10000</td>
<td>1564.8</td>
<td>116 147</td>
</tr>
<tr>
<td>11000</td>
<td>1603.8</td>
<td>127 162</td>
</tr>
<tr>
<td>12000</td>
<td>1621.9</td>
<td>139 176</td>
</tr>
<tr>
<td>13000</td>
<td>1623.4</td>
<td>143 182</td>
</tr>
<tr>
<td>14000</td>
<td>1623.4</td>
<td>143 182</td>
</tr>
<tr>
<td>15000</td>
<td>1623.4</td>
<td>143 182</td>
</tr>
</tbody>
</table>

Please note that values of \( p_i - q_i; i = 1, 2 \) and \( q_i - h_i; i = 1, 2 \) that are shown in Table 3. \( p_i - q_i; i = 1, 2 \) are unit costs of underordering that result from failing to order a unit that could have been sold during the period. \( q_i - h_i; i = 1, 2 \) are unit costs of overordering that result from ordering a unit that could not be sold during the period. Table 3 indicates that Product 1 and Product 2 have the same unit cost of underordering, while Product 1 has more unit cost of overordering than that of Product 2. Referring to Tables 2 and 3, this example suggests that the merchant should order more Product 2 than Product 1 to get more profit.
Table 3: Unit cost of underordering and overordering

<table>
<thead>
<tr>
<th>Product i</th>
<th>$p_i - q_i$</th>
<th>$q_i - h_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

In addition, as shown in Table 2, when storage space constraint increases from 13000 $cm^3$ to 15000 $cm^3$, the optimal solutions do not change. The reason is that as the merchant orders more products, cost of overordering will increase. In this example, if the storage space is greater than 13000 $cm^3$, the optimal order quantities of Product 1 and Product 2 are 143 units and 182 units.

**Example 2.** Suppose that a merchant wants to maximize the total expected profit of three products and set confidence level $\beta = 0.8$. Three different products have the same unit selling price, unit procurement cost and unit salvage value, but they have different uncertain demand and storage space distributions. The associated parameters are listed in Table 4.

Table 4: Parameters of Example 2.

<table>
<thead>
<tr>
<th>Product i</th>
<th>$p_i$ ($/unit$)</th>
<th>$q_i$ ($/unit$)</th>
<th>$h_i$ ($/unit$)</th>
<th>$\xi_i$</th>
<th>$\eta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>3</td>
<td>1</td>
<td>$\mathcal{L}(0, 50)$</td>
<td>$\mathcal{L}(30, 40)$</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>3</td>
<td>1</td>
<td>$\mathcal{L}(0, 200)$</td>
<td>$\mathcal{L}(30, 40)$</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>3</td>
<td>1</td>
<td>$\mathcal{L}(0, 200)$</td>
<td>$\mathcal{L}(5, 15)$</td>
</tr>
</tbody>
</table>

The optimal order quantities and expected profits of Product $i$, $i = 1, 2, 3$ are listed in Table 5. Product 3 has the greatest demand and the least storage space requirement. Hence the example suggests that the merchant should order the most quantity of Product 3. Comparing Product 1 with Product 2, we find that they have the same uncertain storage space distribution, but the Product 1 has less demand than Product 2. Therefore,
the optimal order quantity of Product 1 is less than that of Product 2.

Table 5: Optimal order quantity and expected profit with different storage space constraints-Example 2.

<table>
<thead>
<tr>
<th>Storage space (cm$^3$)</th>
<th>Expected profit ($)</th>
<th>Order quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Product 1</td>
</tr>
<tr>
<td>5000</td>
<td>1158.7</td>
<td>18</td>
</tr>
<tr>
<td>6000</td>
<td>1202.0</td>
<td>22</td>
</tr>
<tr>
<td>7000</td>
<td>1288.3</td>
<td>27</td>
</tr>
<tr>
<td>8000</td>
<td>1223.0</td>
<td>32</td>
</tr>
<tr>
<td>9000</td>
<td>1452.4</td>
<td>37</td>
</tr>
<tr>
<td>10000</td>
<td>1225.0</td>
<td>39</td>
</tr>
<tr>
<td>11000</td>
<td>1225.0</td>
<td>39</td>
</tr>
<tr>
<td>12000</td>
<td>1225.0</td>
<td>39</td>
</tr>
<tr>
<td>13000</td>
<td>1225.0</td>
<td>39</td>
</tr>
<tr>
<td>14000</td>
<td>1225.0</td>
<td>39</td>
</tr>
<tr>
<td>15000</td>
<td>1225.0</td>
<td>39</td>
</tr>
</tbody>
</table>

Moreover, as shown in Table 5, when storage space constraint changes from 10000 cm$^3$ to 15000 cm$^3$, the optimal solutions are the same one. This is because the increasing cost of overordering will cut down the expected profit.

Example 3. In order to examine the sensitivity of confidence level $\beta$, we do experiment on the model by changing confidence level $\beta$. Assume that storage space constraint $V=2000$ cm$^3$. Table 6 lists parameters of example 3.
Table 6: Parameters of Example 3.

<table>
<thead>
<tr>
<th>Product $i$</th>
<th>$p_i$($/unit)</th>
<th>$q_i$($/unit)</th>
<th>$h_i$($/unit)</th>
<th>$\xi_i$</th>
<th>$\eta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>$\mathcal{L}(0, 320)$</td>
<td>$\mathcal{L}(10, 20)$</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>5</td>
<td>2</td>
<td>$\mathcal{L}(0, 300)$</td>
<td>$\mathcal{L}(15, 18)$</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>8</td>
<td>3</td>
<td>$\mathcal{L}(0, 200)$</td>
<td>$\mathcal{L}(25, 30)$</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>12</td>
<td>5</td>
<td>$\mathcal{L}(0, 120)$</td>
<td>$\mathcal{L}(20, 40)$</td>
</tr>
</tbody>
</table>

The computational results are summarized in Fig. 1 that means the expected total profit decreases with $\beta$ increasing from 0 to 1. The example suggests that since the inverse uncertainty distributions of $\eta_i, i = 1, 2, \ldots, n$, are monotone increasing functions on $[0, 1]$, as $\beta$ increases, the feasible region for the model (2) shrinks and the optimal solution becomes smaller.

![Figure 1: The sensitivity of optimal expected profit with different $\beta$ of Example 3.](image)

5. Conclusions

This paper concerns about multi-product newsboy problem, whose quantity demand and storage space of products are uncertain variables. Uncertainty theory provides a new
approach to deal with indeterminacy information. Under the framework of uncertainty theory, we proposed a new multi-product newsboy model to maximize the expected profit under a warehousing chance constraint. It is proved that there exists an equivalence relation between the uncertain multi-product newsboy problem and the integer programming problem in deterministic environment. This equivalence relation leads to the problem can solved by branch-and-bound method to obtain the optimization solution. Finally, three numerical examples were given to illustrate how to use the proposed model to make ordering policy.

Acknowledgements

This work was supported by the National Natural Science Foundation of China Grant No.61273044 and Education Research Project No.YPGC2011-W03.

References


