Bin Packing Problem with Uncertain Volumes and Capacities

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Abstract Bin packing problem is a traditional problem in combinatorial optimization and applied widely in modern society. This paper presents bin packing problem with uncertain volumes and capacities. First, two types of uncertain programming models are mathematically constructed for the uncertain bin packing problem, including expected value model and chance-constrained programming model. Next, the proposed models can be transformed into the corresponding deterministic forms by taking advantage of uncertainty theory. Finally, a numerical experiment is presented to show the performances of the proposed models and algorithms.

Keywords Bin Packing Problem · Uncertainty Theory · Uncertain Variable · Uncertain Programming Model

1 Introduction

Bin packing problem is a historically interesting problem, in which objects of different volumes must be packed into a finite number of bins of fixed capacity in a way that minimizes the number of bins used. The bin packing problem has been widely studied due to its various applications in transportation and logistics industry.

There is an important connection between bin packing and another very important collection of operations research questions. The bin packing problem can also be viewed as a special case of the cutting stock problem (de Carvalho, 2002). When the number of bins is restricted to 1 and each item is characterized
by both a volume and a value, the problem of maximizing the value of items
that can fit in the bin is known as the knapsack problem.

There are many variations of the bin packing problem, such as 2D pack-
ing (Clautiaux et al., 2007; Lodi et al., 2002; Pisinger and Sigurd, 2005), 3D
packing (Martello et al., 2000), multiobjective packing (Liu et al., 2008), on-
line packing (Babel et al., 2004), dynamic bin packing (Coffman et al., 1983;
Epstein and Levy, 2010), multi-stage bin packing (Puchinger and Raidl, 2007),
variable sized packing (Bang-Jensen and Larsen, 2012; Boyar and Favrholdt,
2012; Kang and Park, 2003), and so on.

In computational complexity theory, the bin packing problem is a type
of combinatorial NP-hard problem. Despite the fact that it is NP-hard, op-
timal solutions to very large instances can be produced with sophisticated
algorithms. In many cases, approximation optimal solutions may be obtained
by using efficient heuristic packing algorithms (Johnson, 1974). For example,
the first fit algorithm provides a fast but often non-optimal solution, involving
placing each item into the first bin in which it will fit. The algorithm can be
made much more effective by first sorting the list of elements into decreasing
order (sometimes known as the first-fit decreasing algorithm), although this
still does not guarantee an optimal solution, and for longer lists may increase
the running time of the algorithm. It is known, however, that there always
exists at least one ordering of items that allows first-fit to produce an optimal
solution.

Many evolutionary heuristic algorithms (Coffman et al., 1983), including
genetic algorithms (Chan et al., 2007) and particle swarm algorithms (Liu et
al., 2008), have been developed to find the optimal solutions for various bin
packing problem. Despite the fact that some bin packing problems are NP-
complete, they are amenable to dynamic programming solutions (Han et al.,
2010) or to approximately optimal heuristic solutions (Gendreau et al., 2004;
Stawowy, 2008).

In the above mentioned research, the bin packing problems were investigat-
gated in a deterministic environment, in which the volumes (weights) of the
items, the capacities (costs) of the bins, etc., are positive crisp values. However,
some indeterminacy factors might occur in the problems. In practice, due to
the lack of adequate sample data, or detail sample data are not easy to get be-
cause of economic reasons or technical difficulties, the volumes (weights) of the
items and the capacities (costs) of the bins are not sharply known in advance.
As a result, they are described by some empirical data such as “about 2 cubic
feet” or “approximately 3 cubic meters”. In this situations, it is not suitable to
employ the classical models and algorithms to study the bin packing problem
directly.

Probabilistic approaches are borrowed in the analysis of bin packing al-
gorithm (Foster and Vohra, 1989; Coffman et al., 2008). Moreover, some re-
searchers employed probability theory to investigate the stochastic bin packing
problem (Coffman et al., 1980; Rhee, 1985; Perboli et al., 2012) and considered
the stochastic version of the generalized bin packing problem. Lessmore was
founded in current literature on fuzzy bin packing problem (Kim et al., 2001; Nasibov, 2004).

The indeterminacy phenomena can be divided into distinct types. One is stochastic phenomena, and another is uncertain phenomena. When the sample size is too small or even no-sample to estimate a probability distribution, we have to invite some domain experts to evaluate their belief degree that each event will occur. Since human beings usually overweight unlikely events, the belief degree may have much larger variance than the real frequency. Probability theory is inappropriate in this case because it may lead to counterintuitive results. In order to distinguish from randomness, this phenomenon was named uncertainty (Liu, 2012). In order to deal with the uncertain phenomena, Liu (2007) founded an uncertainty theory, which has become a branch of axiomatic mathematics for modeling human uncertainty. In theoretical aspect, uncertain process (Liu, 2008), uncertain differential equation (Chen and Liu, 2010), have been established. From a practical aspect, uncertain programming (Zhang and Peng, 2012), uncertain network(Zhang and Peng, 2012), uncertain finance (Peng and Yao, 2011), etc., have also been developed.

As an illustration of researching uncertain combinatorial optimization problems, we initially consider uncertain bin packing (UBP) problem, in detail, the bin packing problem with uncertain volumes and uncertain capacities. The conventional bin packing model are extended to uncertain programming models in which the uncertainty distributions of volumes of items and capacities of bins are assumed.

The paper is organized as follows: Section 2 presents some necessary preliminary concepts and results selected from uncertainty theory. In Section 3, the UBP problem is introduced as the extension of classical bin packing problem. In Section 4, two types of uncertain programming models for UBP are presented, which includes expected value model and chance-constrained programming model. Section 5 illustrates an example. The last section contains a summary.

2 Preliminaries

As an efficient tool of modeling the behavior of uncertain phenomena, uncertainty theory is employed to deal with uncertain bin packing problem. Uncertainty theory is a branch of mathematics based on normality, duality, subadditivity, and product axioms. In this section, we present some related preliminaries from uncertainty theory.

The first fundamental concept in uncertainty theory is uncertain measure that is interpreted as the personal belief degree (not frequency) of an uncertain event that may occur.

Let \( \Gamma \) be a nonempty set, and \( \mathcal{L} \) a \( \sigma \)-algebra over \( \Gamma \). For any \( A \in \mathcal{L} \), Liu (2007) presented an axiomatic uncertain measure \( \mathcal{M}(A) \) to express the chance that uncertain event \( A \) occurs. The set function \( \mathcal{M}\{\cdot\} \) satisfies the following three axioms:

1. \( \mathcal{M}\{\emptyset\} = 0 \)
2. \( \mathcal{M}\{\Gamma\} = 1 \)
3. \( \mathcal{M}\{A\cup B\} \leq \mathcal{M}\{A\} + \mathcal{M}\{B\} \)

where \( \emptyset \) is the null set and \( \Gamma \) is the universal set.
(i) (Normality) \( \mathcal{M}\{\Gamma\} = 1; \)
(ii) (Duality) \( \mathcal{M}\{A\} + \mathcal{M}\{A^c\} = 1 \) for any \( A \in \mathcal{L}; \)
(iii) (Subadditivity) For every countable sequence of events \( \{A_i\} \), we have 
\[ \mathcal{M}\{\bigcup_{i} A_i\} \leq \sum_{i} \mathcal{M}\{A_i\}. \]

The triplet \((\Gamma, \mathcal{L}, \mathcal{M})\) is called an uncertainty space.

Liu (2009) defined product uncertain measure by way of the fourth axiom of uncertainty theory, which makes much differences in operations between uncertainty theory and probability theory. Let \((\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)\) be uncertainty spaces for \(k = 1, 2, \ldots\)

Write \( \Gamma = \Gamma_1 \times \Gamma_2 \times \cdots, \mathcal{L} = \mathcal{L}_1 \times \mathcal{L}_2 \times \cdots \)

Then the product uncertain measure \( \mathcal{M} \) on the product \(\sigma\)-algebra \( \mathcal{L} \) is defined by the following axiom:

(iv) (Product Axiom)
\[ \mathcal{M}\left\{ \prod_{k=1}^{\infty} A_k \right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{A_k\} \]
where \(A_k\) are arbitrarily chosen events from \(\mathcal{L}_k\) for \(k = 1, 2, \ldots\), respectively.

An uncertain variable is defined as a measurable function from an uncertainty space to the set of real numbers (Liu, 2007). An uncertain variable \( \xi \) can be characterized by its uncertainty distribution \( \Phi: \mathbb{R} \rightarrow [0, 1] \), which is defined by Liu (2007) as follows
\[ \Phi(x) = \mathcal{M}\{ \gamma \in \Gamma \mid \xi(\gamma) \leq x \}. \]

Let \( \xi \) be an uncertain variable with uncertainty distribution \( \Phi \). Then the inverse function \( \Phi^{-1} \) is called the inverse uncertainty distribution of \( \xi \).

The expected value of uncertain variable \( \xi \) is defined by Liu (2007) as
\[ E[\xi] = \int_{0}^{+\infty} \mathcal{M}\{\xi \geq r\}dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq r\}dr \]
provided that at least one of the two integrals is finite.

As a useful representation of expected value, it has been proved by Liu (2007) that
\[ E[\xi] = \int_{0}^{1} \Phi^{-1}(\alpha)d\alpha \]
where \( \Phi^{-1} \) is the inverse uncertainty distribution of uncertain variable \( \xi \).

Liu (2007) introduced the independence concept of uncertain variables.
The uncertain variables \( \xi_1, \xi_2, \ldots, \xi_m \) are independent if and only if
\[ \mathcal{M}\left\{ \bigcap_{i=1}^{m} \{\xi_i \in B_i\} \right\} = \min_{1 \leq i \leq m} \mathcal{M}\{\xi_i \in B_i\} \]
for any Borel sets \( B_1, B_2, \ldots, B_m \) of \(\mathbb{R} \).
A real-valued function \( f(x_1, x_2, \cdots, x_n) \) is said to be strictly increasing if
\[
f(x_1, x_2, \cdots, x_n) \leq f(y_1, y_2, \cdots, y_n)
\]
whenever \( x_i \leq y_i \) for \( i = 1, 2, \cdots, n \) and
\[
f(x_1, x_2, \cdots, x_n) < f(y_1, y_2, \cdots, y_n)
\]
x \( < y \) for \( i = 1, 2, \cdots, n \).

Liu (2007) introduced the following useful theorem to determine the distribution function of the strictly increasing function of uncertain variables.

**Theorem 1** Let \( \xi_1, \xi_2, \cdots, \xi_n \) be independent uncertain variables with uncertainty distributions \( \Phi_1, \Phi_2, \cdots, \Phi_n \), respectively. If \( f \) is a strictly increasing function, then \( \xi = f(\xi_1, \xi_2, \cdots, \xi_n) \) is an uncertain variable with inverse uncertainty distribution
\[
\Phi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)).
\]

### 3 Uncertain Bin Packing Problem

#### 3.1 Classical Bin Packing Problem

Let us consider the classical bin packing problem first. The bin packing problem is to pack a set of items into a number of bins such that the total capacity does not exceed some maximum value.

Assume that we have \( i \) kinds of items, labelled from 1 to \( m \). Each kind of item \( i \) has a volume \( V_i \), respectively. Of course, all volumes \( V_i \) are nonnegative. Without loss of generality, we can also assume that the items are listed in increasing order of volume in order to simplify the representation. Meanwhile, we have \( j \) kinds of bins, labelled from 1 to \( n \). Each kind of bin \( j \) has a capacity \( C_j \), respectively. Undoubtedly, all capacities \( C_j \) are nonnegative.

The most common formulation of the problem is the 0-1 bin packing problem, which restricts the number \( x_i \) of copies of each kind of item to zero or one. Mathematically, the 0-1 bin packing problem can be formulated as the following bin packing programming

\[
\begin{align*}
\min & \sum_{j=1}^{n} y_j \\
\text{subject to :} & \sum_{i=1}^{m} V_i x_{ij} \leq C_j \cdot y_j, \; \forall j \in \{1, 2, \cdots, n\} \\
& \sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, 2, \cdots, m\} \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \cdots, m\}; \\
& \quad \forall j \in \{1, 2, \cdots, n\} \\
& y_{j} \in \{0, 1\}, \quad \forall j \in \{1, 2, \cdots, n\}.
\end{align*}
\]
Remark 1. \( x_{ij} = 1 \) means that packing item \( i \) into bin \( j \); Otherwise, \( x_{ij} = 0 \). \( y_j = 1 \) means that bin \( j \) be used; Otherwise \( y_j = 0 \). The first constraint requires that the total volume of the items that packed into bin \( j \) must not exceed the capacity of the bin \( j \). The second constraint requires that each item must be packed into exactly one bin.

Specially, the corresponding optimization problem asks for the minimum number of bins of identical capacity \( C \) such that all items can be packed. It might be formulated by the following integer linear programming

\[
\begin{align*}
\min & \sum_{j=1}^{n} y_j \\
\text{subject to:} & \\
& \sum_{i=1}^{m} V_i x_{ij} \leq C \cdot y_j, \forall j \in \{1, 2, \cdots, n\} \\
& \sum_{j=1}^{n} x_{ij} = 1, \forall i \in \{1, 2, \cdots, m\} \\
& x_{ij} \in \{0, 1\}, \forall i \in \{1, 2, \cdots, m\}; \forall j \in \{1, 2, \cdots, n\} \\
& y_j \in \{0, 1\}, \forall j \in \{1, 2, \cdots, n\}.
\end{align*}
\]

3.2 Uncertain Bin Packing Problem

Given a set of items, each of a specific volume, and a set of bins, each of a specific capacities as well—is there a distribution of items to bins such that no item is left unpacked and no bin capacity is exceeded?

The UBP problem is a generalization of the classical bin packing problem and is defined as follows: Given a list of bins with uncertain capacities \( \eta_1, \eta_2, \cdots, \eta_n \), and a list \( \xi_1, \xi_2, \cdots, \xi_m \) of uncertain volumes of the items to pack, the task is to find an integer \( p \) and a partition \( S_1 \cup S_2 \cup \cdots \cup S_p \) of \( 1, 2, \cdots, m \) such that \( \sum_{i \in S_k} \xi_i \leq \eta \) in a sense for all \( k = 1, 2, \cdots, p \). A solution is optimal if it has minimum number \( p \).

Assume that the item \( i \) has an uncertain volume \( \xi_i \) for \( i = 1, 2, \cdots, m \) and the bin \( j \) has an uncertain capacity \( \eta_j \) for \( j = 1, 2, \cdots, n \), respectively. Furthermore, \( \xi_1, \xi_2, \cdots, \xi_m \) are independent uncertain variables. Also, \( \eta_1, \eta_2, \cdots, \eta_n \) are independent uncertain variables.

For simplicity, the uncertain bin packing problem is denoted by \( U = (X, Y, \xi, \eta) \), where \( X = (x_{ij}) \), \( Y = (y_j) \) are decision vectors, \( \xi = (\xi_1, \xi_2, \cdots, \xi_m) \) stands for the uncertain volumes and \( \eta = (\eta_1, \eta_2, \cdots, \eta_m) \) stands for the uncertain capacities.
Formally, the uncertain bin packing problem can be described as

\[
\begin{align*}
\min & \quad \sum_{j=1}^{n} y_j \\
\text{subject to:} & \\
& \sum_{i=1}^{m} \xi_i x_{ij} \leq \eta_j \cdot y_j, \quad \forall j \in \{1, 2, \ldots, n\} \\
& \sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, 2, \ldots, m\} \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \ldots, m\}; \quad \forall j \in \{1, 2, \ldots, n\} \\
& y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \ldots, n\}.
\end{align*}
\]

Since the first constraint is included, the model (3) may be mathematically comprehended in a different way. In order to make it clear, it is inevitable to rank uncertain variables according to some criteria.

### 4 Uncertain Bin Packing Models

#### 4.1 Expected Value Model

In order to rank uncertain variables, expected values can be used as the representation values of different uncertain variables. According to this ranking criterion, the smaller the expected value of uncertain variable is, the smaller the corresponding uncertain variable is.

We present the expected value model for UBP to seek an optimal packing plan as follows:

\[
\begin{align*}
\min & \quad \sum_{j=1}^{n} y_j \\
\text{subject to:} & \\
& E\left[\sum_{i=1}^{m} \xi_i x_{ij}\right] \leq E[\eta_j \cdot y_j], \quad \forall j \in \{1, 2, \ldots, n\} \\
& \sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, 2, \ldots, m\} \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \ldots, m\}; \quad \forall j \in \{1, 2, \ldots, n\} \\
& y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \ldots, n\}.
\end{align*}
\]

Equivalently, it can be rewritten as 0-1 integer programming
\[
\begin{align*}
&\min \sum_{j=1}^{n} y_j \\
&\text{subject to :} \\
&\quad \sum_{i=1}^{m} E[\xi_i]x_{ij} \leq E[\eta_j] \cdot y_j, \forall j \in \{1, 2, \cdots, n\} \\
&\quad \sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, 2, \cdots, m\} \\
&\quad x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \cdots, m\}; \\
&\quad y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \cdots, n\} \\
\end{align*}
\] (5)

In a more obvious way, it can be represented as

\[
\begin{align*}
&\min \sum_{j=1}^{n} y_j \\
&\text{subject to :} \\
&\quad \sum_{i=1}^{m} x_{ij} \int_{0}^{1} \Phi^{-1}_i(\alpha) d\alpha \leq y_j \int_{0}^{1} \Psi^{-1}_j(\alpha) d\alpha, \forall j \in \{1, 2, \cdots, n\} \\
&\quad \sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, 2, \cdots, m\} \\
&\quad x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \cdots, m\}; \\
&\quad y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \cdots, n\} \\
\end{align*}
\] (6)

where \(\Phi^{-1}_i\) and \(\Psi^{-1}_j\) are the inverse uncertainty distributions of \(\xi_i\) and \(\eta_j\), respectively.

4.2 Chance-Constrained Programming Model

Chance-constrained programming is another method to deal with optimal problem in uncertain environment. The basic idea of chance constraint is that it is allowed to violate the constraints, but we need to ensure that the constraints should hold at some chance level.

If the decision maker prefers treating the problem under the chance constraints, we may formulate the UBP as follows:
where $\alpha$ is the predetermined confidence level.

Equivalently, it can be represented as

\[
\begin{align*}
\min & \quad \sum_{j=1}^{n} y_j \\
\text{subject to :} & \\
& \mathcal{M}\{\sum_{i=1}^{m} \xi_i x_{ij} \leq \eta_j \cdot y_j\} \geq \alpha, \quad \forall j \in \{1, 2, \cdots, n\} \\
& \sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in \{1, 2, \cdots, m\} \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \cdots, m\}; \\
& y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \cdots, n\}
\end{align*}
\]

where $\Phi_i^{-1}$ and $\Psi_j^{-1}$ are the inverse uncertainty distributions of $\xi_i$ and $\eta_j$, respectively.

5 Numerical Experiment

In this section, we give a numerical example to show the applications of the models as mentioned above. For the convenience of description, we summarize the problem as follows. Suppose that there are twelve items, each of a specific volume, and six bins, each of a specific capacities as well. Our goal is to make a packing plan such that no item is left unpacked and no bin capacity is exceeded. A plan is optimal if it has minimum number of bins. Before making the plan, the decision maker needs to know the volumes of the items and the capacities of the bins. However, due to economic reason or technical difficulties, usually the decision maker cannot get these data exactly. In this case, we have to
Table 1 The volumes of the items

<table>
<thead>
<tr>
<th>item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td>(3, 4, 5)</td>
<td>(5, 6, 7)</td>
<td>(4, 5, 6)</td>
<td>(4, 6, 8)</td>
<td>(4, 5, 6)</td>
<td>(6, 7, 8)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>item</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume</td>
<td>(3, 4, 5)</td>
<td>(2, 4, 6)</td>
<td>(7, 8, 9)</td>
<td>(5, 6, 7)</td>
<td>(4, 5, 6)</td>
<td>(4, 6, 8)</td>
</tr>
</tbody>
</table>

Table 2 The capacities of the bins

<table>
<thead>
<tr>
<th>bin</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>capacity</td>
<td>(10, 12, 14)</td>
<td>(16, 18, 20)</td>
<td>(18, 19, 20)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>bin</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>capacity</td>
<td>(20, 24, 28)</td>
<td>(22, 24, 26)</td>
<td>(25, 26, 27)</td>
</tr>
</tbody>
</table>

obtain the uncertain data by means of empirical estimation. Assume that all uncertain variables are zigzag uncertain variables, which are listed in Tables 1 and 2, respectively.

Then the expected value model for the UBP is equivalent to the following model:

\[
\begin{aligned}
\min & \sum_{j=1}^{6} y_j \\
\text{subject to:} & \\
& \sum_{i=1}^{12} x_{ij} \int_{0}^{1} \Phi^{-1}_i(\alpha) d\alpha \leq y_j \int_{0}^{1} \Psi^{-1}_j(\alpha) d\alpha, \forall j \in \{1, 2, \cdots, 6\} \hspace{1cm} (9) \\
& \sum_{j=1}^{6} x_{ij} = 1, \forall i \in \{1, 2, \cdots, 12\} \\
& x_{ij} \in \{0, 1\}, \forall i \in \{1, 2, \cdots, 12\}; \forall j \in \{1, 2, \cdots, 6\} \\
& y_j \in \{0, 1\}, \forall j \in \{1, 2, \cdots, 6\}.
\end{aligned}
\]

We can use mathematical software (e.g., LINGO) to solve this 0-1 integer programming problem. Here the optimal solution of the model is:

\[ x_{15} = x_{24} = x_{36} = x_{46} = x_{55} = x_{65} = x_{74} = x_{86} = x_{96} = x_{10,4} = x_{11,4} = x_{12,5} = 1; y_4 = y_5 = y_6 = 1. \]

Figure 1 states the corresponding optimal packing plan for the problem.
To put it in another way, we can also solve this problem by using chance-constrained programming model. In this case, if we set \( \alpha = 0.9 \), then the chance-constrained programming model for the UBP is equivalent to the following model:

\[
\begin{align*}
\text{min} & \quad \sum_{j=1}^{6} y_j \\
\text{subject to :} & \\
& \sum_{i=1}^{12} \Phi_{i}^{-1}(0.9) x_{ij} - \Psi_{j}^{-1}(0.1) y_j \leq 0, \forall j \in \{1, 2, \cdots, 6\} \\
& \sum_{j=1}^{6} x_{ij} = 1, \quad \forall i \in \{1, 2, \cdots, 12\} \\
& x_{ij} \in \{0, 1\}, \quad \forall i \in \{1, 2, \cdots, 12\}; \\
& \quad \forall j \in \{1, 2, \cdots, 6\}; \\
& y_j \in \{0, 1\}, \quad \forall j \in \{1, 2, \cdots, 6\}. 
\end{align*}
\]

(10)

Solved by mathematics soft, the optimal solution of the model is:

\[ x_{14} = x_{23} = x_{35} = x_{45} = x_{53} = x_{66} = x_{74} = x_{83} = x_{94} = x_{10,6} = x_{11,6} = x_{12,5} = 1; \ y_3 = y_4 = y_5 = y_6 = 1. \]

Figure 2 states the corresponding optimal packing plan for the problem.
6 Conclusion

Bin packing problem is one of many optimization problems which are of both theoretical and applied interest in mathematics. The uncertain bin packing problem arises from various applications in real life. Uncertainty theory plays the key role of mathematical model to deal with uncertain bin packing problem.

The main contributions are three-fold listed as follows: Firstly, two new types of models were proposed for uncertain bin packing problem with different modeling ideals, that is, expected value model and chance-constrained programming model are proposed for the problem. Secondly, these models can be turned into the deterministic forms by taking advantage of properties of uncertainty theory. Thirdly, a numerical example was also presented to show the applications of the models.

It is important to keep in mind that these models are constructed from the different points of view. We cannot conclude which model is better in the process of decision making. The application of different models is completely based on the decision maker’s preference.

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