Uncertain Programming Models for Sports Supplier Selection with Cost Minimization

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Abstract: In this paper, we consider three types of uncertain programming models for the purpose of selecting appropriate suppliers and allocating order quantities to them under uncertain environment. The decision objective is to minimize the total cost incurred in the whole process, which consists of purchasing cost, holding cost and ordering cost. This paper chooses the case with multiple sources, multiple criteria and capacity constraints with uncertain parameters. In order to solve the proposed models, hybrid intelligent algorithm which integrating genetic algorithm by uncertain simulation is designed.

Keywords: Supplier selection; Uncertain variable; Uncertain programming; Genetic algorithm

1 Introduction

For manufacturers, external purchase is a substantial expenditure, which takes up an important part of the value of products. In today’s business world, outsourcing to external suppliers becomes a hot topic. For industrial companies, purchasing’s share in the total turnover customarily ranges between 50% and 90% (Telgen [16]). To our knowledge, several works have taken into multiple objective supplier selection account. These literature includes Dickson [3] and Weber et al [18], in which objectives include net price, quality, delivery, performance history, capacity, communication system, service and so forth. The main aim is to choose appropriate suppliers which perform optimally on the desired dimensions.

The developments in supply chain management offer the opportunity for selecting suppliers with the minimal cost of ownership associated with purchasing process. The method goes beyond price to consider all costs over the goods’ whole life including quantities related to service, quality, delivery, administration, communication, failure, maintenance and so forth (Ellram [4][5]). For large companies with cost accounting systems, Timmerman [17] introduces the cost-ratio method to collect the costs related to quality, delivery and service. Monczka & Trecha [9] and Smytka & Clemens [14] employ a total cost approach with rating systems for criteria such as service and delivery performance.

Uncertainty and imprecision are inherent in real life. Several scholars dealt with uncertainty in their works. Soukoup [15] introduced a simulation-based approach to handle uncertainty in the demand or service purchased. The analytic hierarchy process is formulated to handle imprecision in supplier selection problem (Nydick & Hill [11], Barbarosoglu & Yazgac [1] and Narasimhan [10]). Ronen & Trietsch [13] developed a decision support system for supplier selection where the order lead time is assumed to be uncertain.

Fuzziness is another uncertainty in real life, and fuzzy methods have been applied to the field of finance and management. In particular, the quantities e.g. demand in supply chain management can be obtained by experts as fuzzy variables when there is no or not enough data. There are several works contributed to supply chain management in fuzzy environment by using credibility theory to deal with fuzziness, e.g. Qin and Ji [12] proposed a logistics network with product recovery in fuzzy environment. In order to model other imprecise quantities, Liu [8] presented uncertainty theory to describe human decisions in the state of uncertainty. Thus, we assume that the uncertain parameters are uncertain variables.

In this paper, we employ uncertain programming techniques to account for supplier selection problem in uncertain environment. The objective function is to minimize the total cost including order cost, holding cost and purchasing cost, and the capacity as the constraint. Long-term cooperation between manufacturers and suppliers is typically more important in the manufactory process. A minimum ordering rate is introduced to set in order to keep a relationship with all the suppliers. We assume that all the suppliers need be given the allocation at the minimum rate. The demand of the manufacturer is confirmed by the retailers. The behavior of the manufacturer is ahead of that of retail. The manufactory cannot obtain a precise amount when choosing the suppliers. Finally, we employ uncertain programming methods to model supplier selection problem. Considering the complexity, the traditional algorithms are impossible to be applied to solve the models. Therefore, we use so-called 99-method to solve the proposed models.

The rest of the paper is organized as follows. In Section 3, we briefly introduce the knowledge of uncertainty theory. Section 3 lists some symbols and notations used in this paper. Three optimization models named expected value model, α-cost minimization model and chance maximization model are established for supplier selection problem in Section 4.
Section 5 introduces 99-method which can be used to numerically solve proposed models. Finally, remarking conclusions are listed in Section 6.

2 Preliminaries

Let \( \Gamma \) be a nonempty set, and let \( A \) be a \( \sigma \)-algebra over \( \Gamma \). Each element of \( A \) is called an event. A set function is called an uncertain measure \([8]\) if and only if it satisfies the following four axioms:

Axiom 1. (Normality) \( M[\Gamma] = 1 \);
Axiom 2. (Monotonicity) \( M[A] \leq M[B] \) whenever \( A \subseteq B \);
Axiom 3. (Self-Duality) \( M[A] + M[A^c] = 1 \) for any event \( A \);
Axiom 4. (Countable Subadditivity) \( M(\bigcup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} M[A_i] \) for any countable sequence of events \( A_i \).

Definition 2.1 (Liu [8]) Let \( \Gamma \) be a nonempty set, and let \( A \) be a \( \sigma \)-algebra over it. If \( M \) is an uncertain measure \([8]\) if and only if it satisfies the following four axioms:

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Definition 2.2 (Liu [8]) Uncertain variable \( \xi \) is defined as a measurable function from an uncertainty space \((\Gamma, A, M)\) to the set of real numbers \( \mathbb{R} \). That is, for any Borel set \( B \), we have

\[ \{ \gamma \in \Gamma | \xi(\gamma) \in B \} \in A. \]

Definition 2.3 (Liu [8]) Let \( \xi \) be an uncertain variable. Then the expected value of \( \xi \) is defined as

\[ E[\xi] = \int_{0}^{\infty} M[\xi \geq x] dx - \int_{\infty}^{0} M[\xi \leq x] dx \]

provided that at least one of the two integrals is finite.

Definition 2.4 (Liu [8]) Let \( \xi \) be an uncertain variable, and \( \alpha \in (0, 1) \). Then

\[ \xi_{\inf}(\alpha) = \inf\{ r | M[\xi \leq r] \geq \alpha \} \]

is called the \( \alpha \) – pessimistic value of \( \xi \).

Theorem 2.1 (Linearity of Expected Value Operator, Liu [8]) Let \( \xi \) and \( \eta \) be independent uncertain variables with finite expected values. Then for any real numbers \( a \) and \( b \), we have

\[ E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \]

3 Problem Formulation

Supplier selection problem concerns with how to choose appropriate suppliers and how to assign appropriate quantities to perform optimally for the desired purpose. Here, we list several symbols and notations which will be used in the rest of the paper. \( \xi \) is the annual demand (uncertain variable). \( Q \) is the ordered quantity to all suppliers in each period. \( H \) represents the inventory holding cost. \( n \) is the number of suppliers. \( O_i \) represents the ordering cost of the supplier \( i = 1, 2, \ldots, n \) each period. \( C_i \) is the price of supplier \( i = 1, 2, \ldots, n \). \( S_i \) is the capacity of supplier \( i = 1, 2, \ldots, n \). \( R \) is the minimum ordering quantity. \( D_i \) is the uncertain percentage of delayed delivery of supplier \( i = 1, 2, \ldots, n \). \( P_i \) represents the minimum accepted quality rate. \( X_i \) is the percent of \( Q \) assigned to supplier \( i = 1, 2, \ldots, n \) (decision variable).

We suppose that a buyer wants to give more quantities to the best supplier and gives minimum rate to the bad supplier from among the \( n \) vendors whose capacities are fixed. The buyer wants to minimize the annual total cost (C) including annual purchasing costs (P), ordering costs (O) and holding costs (H), subject to limitations on his/her budget, quality, service etc. In addition, the buyer needs to consider minimum quality rate, maximum delayed delivery and other constraints.

Annual Purchasing Cost (C)

We suppose the purchased quantities from the supplier \( i \) are \( X_i \xi \) and the unit price is \( C_i \), the total purchasing cost \( (C) \) is

\[ C(x, \xi) = \sum_{i=1}^{n} C_i X_i \xi. \]  

Annual Ordering Cost (O)

To keep relationship with all the suppliers, The suppliers should be paid ordering cost each period. Therefore, the annual ordering cost is

\[ O = \frac{\xi}{Q} \sum_{i=1}^{n} O_i. \]  

Annual Holding Cost (H)

For supplier \( i \), his/her average inventory is \( X_i \xi/2 \) and the inventory holding cost is \( H C_i X_i \). Further, the holding cost is given as follows:

\[ H(x) = \sum_{i=1}^{n} (X_i Q/2) H C_i X_i = \frac{QH}{2} \sum_{i=1}^{n} X_i^2 C_i. \]  

Annual Total Cost (T)

Based on equations (1), (2) and (3), we may derive the following annual total cost

\[ C(x, \xi) = P + O + H = \sum_{i=1}^{n} C_i X_i \xi + \frac{\xi}{Q} \sum_{i=1}^{n} O_i + \frac{QH}{2} \sum_{i=1}^{n} X_i^2 C_i. \]  

It is worthwhile pointing out that both \( P \) and \( O \) are uncertain variables, since \( \xi \) is an uncertain variable. Further, the annual total cost is also an uncertain variable. Considering \( Q \) is the optimal order quantity, therefore, it can be derived by using the derivative of \( C \). It follows from

\[ Q = \sqrt{\frac{2\xi(\gamma) \sum_{i=1}^{n} O_i}{H \sum_{i=1}^{n} X_i^2 C_i}}. \]
that
\[
C(x, \xi) = \sum_{i=1}^{n} c_i x_i \xi + \sqrt{2\xi(\gamma) H(\sum_{i=1}^{n} O_i)(\sum_{i=1}^{n} X_i^2 C_i)}.
\]

\[
(6)
\]

### 3.1 Constraints

Because of lack of resources, the supplier selection problem has several constraints, for example,

- Capacity constraint: \( X_i \xi \leq S_i \);
- Demand constraint: \( \sum_{i=1}^{n} X_i = 1 \);
- Quality constraint: \( \sum_{i=1}^{n} X_i P_i \geq P \);
- Delayed delivery constraint: \( \sum_{i=1}^{n} X_i D_i \leq D \);
- Other constraint: \( R \leq X_i \leq 1 \).

Since the demand \( \xi \) is an uncertain variable, \( X_i \xi \leq S_i \) will become meaningless to compare a function with a real number. Uncertainty theory is useful to deal with this problem, and to formulate it as follows,

\[
M\{\gamma \in \Gamma | X \cdot \xi(\gamma) \leq S_i \} \geq \alpha.
\]

Furthermore, the constraint may be formulated as follows,

\[
\begin{align*}
M\{\gamma \in \Gamma | X \cdot \xi(\gamma) \leq S_i \} &\geq \alpha \\
\Leftrightarrow M\{\gamma \in \Gamma | \xi(\gamma) \leq S_i/X \} &\geq \alpha \\
\Leftrightarrow S_i/X &\geq L(\alpha) \Leftrightarrow X \leq S_i/L(\alpha)
\end{align*}
\]

where \( L(\alpha) = \{L|M\{\gamma \in \Gamma | \xi(\gamma) \leq L \} = \alpha\} \). We may obtain the crisp expressions of the value \( L(\alpha) \) for several special uncertain variables, e.g. triangular, trapezoidal, normal uncertain variables and so on.

### 4 Mathematical Models

It is meaningless to minimize an uncertain variable. If we want to rank uncertain variables, we need to look for some indices such as expected value, critical values and so on. In this section, we employ uncertain programming techniques to model supplier selection problem. By using different criteria, we respectively propose three kinds of models including the expected value model, \(\alpha\)-cost minimization model and chance maximization model.

#### 4.1 Expected Value Model

The most used model is expected value model whose idea is to optimize the expected value of \( C(x, \xi) \) subject to some expected constraints. Here, we formulate the expected value model for supplier selection with uncertain demand as follows:

\[
\min E[C(x, \xi)]
\]

subject to:

\[
\begin{align*}
X_i \cdot E[\xi] &\leq S_i, i = 1, 2, \ldots, n \\
\sum_{i=1}^{n} X_i = 1 \\
\sum_{i=1}^{n} X_i P_i &\geq P \\
\sum_{i=1}^{n} X_i D_i &\leq D \\
R &\leq X_i \leq 1, i = 1, 2, \ldots, n.
\end{align*}
\]

#### 4.2 \(\alpha\)-Cost Minimization Model

Charnes & Cooper [2] initiated chance-constrained programming as a tool to model random decision systems. The main idea of chance-constrained programming is to optimize certain critical value at a predetermined confidence level given several chance constraints. According to the idea, \(\alpha\)-cost minimization model for supplier selection with uncertain demand is proposed as follows:

\[
\min C
\]

subject to:

\[
\begin{align*}
M\{\gamma \in \Gamma | C(x, \xi) \leq C \} &\geq \alpha \\
X_i &\leq S_i/L(\alpha), i = 1, 2, \ldots, n \\
\sum_{i=1}^{n} X_i = 1 \\
\sum_{i=1}^{n} X_i P_i &\geq P \\
\sum_{i=1}^{n} X_i D_i &\leq D \\
R &\leq X_i \leq 1, i = 1, 2, \ldots, n.
\end{align*}
\]

#### 4.3 Chance Maximization Model

In some situations, decision makers want to maximize the chance of the event \( C(x, \xi) \leq C^0 \). In order to describe this case, Liu [6][7] presented the dependent-chance programming. In this subsection, we employ the uncertain dependent-chance programming technique into the supplier selection area as follows:

\[
\max M\{\gamma \in \Gamma | C(x, \xi) \leq C^0 \}
\]

subject to:

\[
\begin{align*}
X_i &\leq S_i/L(\alpha), i = 1, 2, \ldots, n \\
\sum_{i=1}^{n} X_i = 1 \\
\sum_{i=1}^{n} X_i P_i &\geq P \\
\sum_{i=1}^{n} X_i D_i &\leq D \\
R &\leq X_i \leq 1, i = 1, 2, \ldots, n.
\end{align*}
\]
5 Hybrid Intelligent Algorithm

In Section 3, three models all contain uncertain parameter so that it is to solve these models by using classical methods. In order to solve the problem, Liu [8] proposed a 99-method to calculate the relative items. Here note that there is no difference between deterministic mathematical programming and uncertain programming except for that the latter has uncertain functions. Essentially, there are three types of uncertain functions in the above models,

\[ U_1: x \to E[C(x, \xi)] \]
\[ U_2: x \to M\{C(x, \xi) \leq C^0\} \]
\[ U_3: x \to \min\{C|M\{C(x, \xi) \leq C\} \geq \alpha\} \]

Note that those uncertain functions may be calculated by the 99-method if the function C is monotone. In fact, it is easy to see that \( C(x, \xi) \) is an increasing function of \( \xi \).

**Lemma 5.1** (Liu [8]) Let \( \xi \) be an uncertain variable with uncertainty distribution \( \Phi \), and let \( f \) be a strictly decreasing function. Then \( f(\xi) \) is an uncertainty distribution with inverse uncertainty distribution

\[ \Psi^{-1}(\alpha) = f(\Phi^{-1}(1 - \alpha)). \]  

Based on this theorem, Liu [8] presented the following operational law.

**99-Method** Let \( \xi \) be an uncertain variable represented by a 99-table. Then for any strictly decreasing function \( f(x) \), the uncertain variable \( f(\xi) \) has a 99-table,

| 0.01 | 0.02 | 0.03 | \cdots | 0.99 |
| --- | --- | --- | \cdots | --- |
| \( x_1 \) | \( x_2 \) | \( x_3 \) | \cdots | \( x_{99} \) |

According the above 99-method, we can compute the discrete uncertainty distribution of \( C(x, \xi) \) as the approximation. Then these three uncertain functions \( U_1, U_2 \) and \( U_3 \) can be obtained. Then we can find a numerical method for solving deterministic mathematical programming, for example, genetic algorithm, particle swarm optimization, or any classical algorithms. Then, for example, we may integrate the 99-method and the genetic algorithm to produce a hybrid intelligent algorithm for solving the above uncertain programming models:

**Step 1.** Initialize \( pop_{size} \) chromosomes whose feasibility may be checked by the 99-method.

**Step 2.** Calculate all the objective values for all chromosomes.

**Step 3.** Compute the fitness of all chromosomes.

**Step 4.** Select the chromosomes by spinning the roulette wheel.

**Step 5.** Renew the chromosomes by spinning the roulette wheel.

**Step 6.** Repeat the second to the fifth steps for a given number of cycles.

**Step 7.** Report the best chromosome as the optimal solution.

6 Conclusion

This paper used uncertain programming techniques to model supplier selection problem with uncertain demand. According to different criteria, expected value model, \( \alpha \)-cost minimization model and chance maximization model were respectively established to choose the desired suppliers. These models have the following characteristics: a. the total cost is composed of purchasing cost, ordering cost and holding cost; b. keeping long-term relationship with the suppliers is considered; c. three uncertain programming models may be used to solve different cases.

References


