Abstract: With the developments of logistics management, product recovery has been paid more attentions in the past decades. However, the currently available literature focuses on deterministic environment. In fact, uncertainty is one of the characteristics of product recovery network. In order to model this case, we consider a generalized product recovery network with uncertain quantities of returned goods and uncertain transportation costs. By using different criteria, we formulate three uncertain stochastic programming models for this problem.

Keywords: Uncertain programming, Uncertain variable, Product recovery, Uncertainty theory

1 Introduction

There are two driving forces to develop a product recovery network. One is from economics including direct and indirect gains, e.g. a company may reduce the cost and add the value by returning used products. Meanwhile, the company can indirectly improve market competitiveness and service level. The second reason is environmental factors such as environmental laws and environmental consciousness of consumers. Several countries have charged producers with responsibility for the whole product life, such as European countries gradually increase environmental legislations including recycling quotas, manufacturing take-back responsibility. Thus, product recovery has now received more and more attentions.

Designing a reasonable network is an important issue of prime importance for economic viability of goods recovery activities. Some large-scale businesses are considering the problem. At present, a growing number of literatures are contributed to network design models for returned product. Del Castillo and Cochran [1] introduced a couple of linear programming to solve optimal short horizon distribution operations for reusable container systems. Jayaraman et al. [5] presented two mathematical formulations for reverse distribution logistical problems that contains goods recall, goods recycling and reuse, goods disposal, and hazardous goods return. Fleischmann et al. [4] review quantitative models for product recovery problem. Min et al. [12] establish a nonlinear mixed-integer programming to model a multi-echelon product recovery network.

Uncertainty is one of the characteristics of product recovery network. Different from the traditional forward logistics, it is impossible to estimate the timing and quantity of returned goods and the operating costs. Up to now, the most existing models only deal with deterministic quantities. In order to deal with uncertainty, some researchers regarded the relative parameters as stochastic quantities and set up mathematical models which take random information into account. A stochastic programming approach was employed to model a deterministic location model for goods recovery problem in Listes and Dekker [8]. A mixed integer programming for a generic goods recovery network with capacity limits, multi-product management was considered in Salema et al. [14]. They assumed goods demands and returns are both random variables, and they used scenario-based approach to model the problem. In addition, Lieckens and Vandaele [7] extended a mixed integer linear program models with some queueing characteristics using a G/G/m model and the random lead times were considered. In addition, several researchers investigate product recovery network design problem in fuzzy environment such as Qin and Ji [13].

Different from the above works, in this paper, we assume that returned quantities are uncertain variable, and we consider a product recovery network under uncertain environment. More specifically, we formulate three types of stochastic programming models for a generalized but more realistic product recovery network. The rest of the paper is structured as follows. Section 2 recall uncertainty theory. A detailed problem description is given in Section 3. Section 4 presents three types of uncertain programming models based on different criteria.

2 Preliminaries

In this section, we briefly introduce the basic of uncertainty theory. Let \( \Gamma \) be a nonempty set, and let \( \mathcal{A} \) be a \( \sigma \)-algebra over \( \Gamma \). Each element of \( \mathcal{A} \) is called an event. A set function is called an uncertain measure [10] if and only if it satisfies the following four axioms

Axiom 1. (Normality) \( \mathcal{M}\{\Gamma\} = 1; \)
Axiom 2. (Monotonicity) $\mathcal{M}\{A\} \leq \mathcal{M}\{B\}$ whenever $A \subseteq B$;
Axiom 3. (Self-Duality) $\mathcal{M}\{A\} + \mathcal{M}\{A^c\} = 1$ for any $A$;
Axiom 4. (Countable Subadditivity) $\mathcal{M}\left\{ \bigcup_{i=1}^{\infty} A_i \right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{A_i\}$ for any countable sequence of events $\{A_i\}$.

Definition 2.1. (Liu [10]) Let $\Gamma$ be a nonempty set, and let $A$ be a $\sigma$-algebra over it. If $\mathcal{M}$ is an uncertain measure, then the triplet $(\Gamma, A, \mathcal{M})$ is called an uncertainty space.

Definition 2.2. (Liu [10]) Uncertain variable $\xi$ is defined as a measurable function from an uncertainty space $(\Gamma, A, \mathcal{M})$ to the set of real numbers $\mathbb{R}$. That is, for any Borel set $B$, we have $\{ \gamma \in \Gamma \mid \xi(\gamma) \in B \} \in A$.

Definition 2.3. (Liu [10]) Let $\xi$ be an uncertain variable. Then the expected value of $\xi$ is defined as

$$E[\xi] = \int_{-\infty}^{\infty} \mathcal{M}\{\xi \geq x\} dx - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq x\} dx$$

provided that at least one of the two integrals is finite.

Definition 2.4. (Liu [10]) Let $\xi$ be an uncertain variable, and $\alpha \in (0, 1]$. Then

$$\xi_{\alpha}(\alpha) = \inf \{ r \mid \mathcal{M}\{\xi \leq r\} \geq \alpha \}$$

(1)
is called the $\alpha$-pessimistic value of $\xi$.

Theorem 2.1. (Liu [10]) Let $\xi$ and $\eta$ be independent uncertain variables with finite expected values. Then for any real numbers $a$ and $b$, we have $E[a\xi + b\eta] = aE[\xi] + bE[\eta]$.

3 Problem Description

A product recovery network is composed of four entities: consumers, collection sites, disposal centers, and plants. In order to take advantage of economies of scale, returned goods are first transported to the collection sites. These sites may be the warehouse of retailer or wholesaler of the forward logistics. At the end of each collection period, returned goods are transported to some disposal center to sort and handle goods. After being handled, the useful material is delivered to a plant producing new products by raw or returned material. The aim of network is to decide the location and allocation of the entities and returned flows between them with minimal total cost to meet consumers’ objective.

3.1 Assumptions

1. The collection site has sufficient capacity to accommodate returned goods during the collection period.
2. Consumers return the used goods to a collection center and cannot directly return them to the disposal center or plant due to insufficient volume.
3. Each consumer chooses a collection site whose distance doesn’t exceed a given distance constant, and transportation costs as well as their collection sites are ignored.

3.2 Symbols

1. Indices:
   - $i$: index for consumers, $i = 1, 2, \ldots, I$
   - $j$: index for collection sites, $j = 1, 2, \ldots, J$
   - $k$: index for disposal centers, $k = 1, 2, \ldots, K$
   - $l$: index for plants, $l = 1, 2, \ldots, L$
2. Parameters:
   - $\xi_i$: daily volume of goods returned by consumer $i$ (uncertain variable)
   - $f_j$: annual cost of opening collection site $j$
   - $g_k$: fixed cost for establishing disposal center $k$
   - $h_l$: fixed cost for establishing plant $l$
   - $s$: per unit daily inventory carrying cost
   - $\gamma$: unit penalty cost for non-collected returned volumes
   - $\lambda_k$: unit handling cost in disposal center $k$
   - $\eta_{jk}$: unit transportation cost from collection site $j$ to disposal center $k$ (uncertain variable)
   - $\zeta_{kl}$: unit transportation cost from disposal center $k$ to plant $l$ (uncertain variable)
   - $d_{ij}$: distance from consumer $i$ to collection site $j$
   - $d_{jk}$: distance from collection site $j$ to disposal center $k$
   - $d_{lk}$: distance from disposal center $k$ to plant $l$
   - $V_k$: maximum capacity of disposal center $k$
   - $M_l$: maximum capacity of plant $l$
   - $\rho$: ration of reused materials of returned goods
   - $w$: total working days of one year
   - $T_1$: length of a collection period at each collection site
   - $T_2$: length of a period in disposal center
   - $q, Q$: minimum, maximum of opening collection sites
   - $r, R$: minimum, maximum of opening disposal centers
   - $s, S$: minimum, maximum number of opening plants
3. Decision variables:
   - $X_{jk}$: returned goods from site $j$ to disposal center $k$
   - $Y_{kl}$: valued material from disposal center $k$ to plant $l$
   - $P_{ij}$: 1, if consumer $i$ is assigned to site $j$
   - $Q_{ij}$: 1, if a collection site $j$ is opened
   - $R_{lk}$: 1, if disposal center $k$ is opened
   - $S_l$: 1, if plant $l$ is opened

3.3 Cost Structure

The total cost includes four parts: setup cost of collection sites, disposal centers, plants; inventory carrying cost in collection sites; handling cost in disposal centers; penalty cost of not satisfying requirements of consumers; transportation cost of shipping returned goods from collection sites to disposal centers and shipping reused materials from disposal centers to plants. Denote $\xi = \{\xi_i\}$, $\eta = \{\eta_{jk}\}$, $\zeta = \{\zeta_{kl}\}$.
proposes the following expected cost minimization model:

\[ \text{min } E[C(X, Y, \xi, \eta, \zeta)] \]

subject to some expected constraints. By using the idea, we

The most popular model of uncertain programming is ex-

4.1 Expected Cost Minimization Model

In real life, it is impossible or difficult to exactly foretell the

hence experts. We assume that returned volume and trans-

In this model, \( X_j \) denotes returned goods

4 Uncertain Programming Models

In real life, it is impossible or difficult to exactly foretell the

returned volumes by consumers and transportation costs. As

uncertain variables. According to the criteria of uncertain

4.2 Chance-constrained Model

Charnes and Cooper [2] proposed chance-constrained pro-

gramming to model a stochastic decision systems. After that,

Liu and Iwamura [11] generalized chance-constrained pro-

gramming for fuzzy environment. Next, we establish chance-

constrained model of uncertain programming for product re-

covery network as follows:

\[ \text{min } f \]

s.t. \( \mathcal{M}(C(X, Y, \xi, \eta, \zeta) \leq f) \geq \beta \)

\[ \mathcal{M}\{ \sum_{i=1}^{J} \xi_i Y_{ij} \geq \sum_{k=1}^{K} X_{jk} \} \leq \alpha, \ j = 1, 2, \ldots, J \]

\[ \rho \cdot \left( \sum_{i=1}^{J} X_{jk} \right) = \sum_{l=1}^{L} Y_{lj}, \ k = 1, 2, \ldots, K \]

\[ \sum_{j=1}^{J} Y_{ij} = 1, \ i = 1, 2, \ldots, I \]

\[ \sum_{j=1}^{J} X_{jk} \leq M \cdot Y_{ij}, \ j = 1, 2, \ldots, J \]

\[ \sum_{j=1}^{J} X_{jk} \leq M \cdot Y_{ij}, \ j = 1, 2, \ldots, J \]

\[ q \cdot \left( \sum_{j=1}^{J} P_{ij} \right) \leq l, \ i = 1, 2, \ldots, I, \ j = 1, 2, \ldots, J \]

\[ r \cdot \sum_{k=1}^{K} R_k \leq R \]

\[ s \leq \sum_{l=1}^{L} S_l \leq S \]

\[ X_{jk}, Y_{ij} \geq 0, \ P_{ij}, Q_{ij}, R_k, S_l \in \{0, 1\} \]

The objective function is to minimize expected value of the total cost. Constraint (1) implies that returned goods

are collected in expected sense. Constraint (2) means that reusable materials after being handled are delivered to plants. All consumers are all assigned to some collection site by con-

straint (3). Constraint (4) prohibits any flows from unopened collection site. Constraints (5) and (6) are respectively the ca-

pacity limit of each disposal and plant. Constraint (7) implies that collection sites should be located within certain allowable proximity of consumer zones. Constraints (8), (9) and (10) limits minimum number and maximum number of collection sites, disposal centers and plants. Constraint (11) preserves the non-negativity of decision variables \( X \) and \( Y \), and enforces the binary restriction of decision variables \( P_{ij}, Q_{ij}, R_k, S_l \).

4.3 Measure Maximization Model

If decision-makers want to maximize the uncertainty mea-

sure of several events simultaneously appearing in a complex

system, then dependent-chance programming may be used to

model this case (see Liu [10]). Therefore, we proposes a new

\[ \mathcal{M}\{ \sum_{i=1}^{I} \eta_{ij} Y_{ij} \geq \sum_{k=1}^{K} X_{jk} \} \leq \alpha, \ j = 1, 2, \ldots, J \]

\[ \sum_{j=1}^{J} X_{jk} \leq M \cdot Y_{ij}, \ j = 1, 2, \ldots, J \]

\[ \sum_{j=1}^{J} X_{jk} \leq M \cdot Y_{ij}, \ j = 1, 2, \ldots, J \]

\[ \sum_{j=1}^{J} X_{jk} \leq M \cdot Y_{ij}, \ j = 1, 2, \ldots, J \]

\[ q \cdot \left( \sum_{j=1}^{J} Q_{ij} \right) \leq l, \ i = 1, 2, \ldots, I, \ j = 1, 2, \ldots, J \]

\[ r \cdot \sum_{k=1}^{K} R_k \leq R \]

\[ s \leq \sum_{l=1}^{L} S_l \leq S \]

\[ X_{jk}, Y_{ij} \geq 0, \ P_{ij}, Q_{ij}, R_k, S_l \in \{0, 1\} \]

\[ (2) \]

In this model, \( \alpha \) and \( \beta \) are given confidence levels set by decision

makers. The first implies the uncertainty measure of the total cost less than or equal to some given value is greater than the given confident level \( \beta \). The second constraint implies that the uncertainty measure of not returning goods returned by consumers is less than or equal to the given confident level.
model for product recovery network with uncertain parameters. The objective is to maximize uncertainty measure of the event of total cost less than or equal to some given cost. Therefore, we establish the following measure maximization model as follows:

\[
\begin{align*}
\min \quad & \mathcal{M}\{C(X, Y, \xi, \eta, \zeta) \leq C_0\} \\
\text{s.t.} \quad & \mathcal{M}\left\{\sum_{i=1}^{I} \xi Y_{ij} \geq \sum_{k=1}^{K} X_{jk}\right\} \leq \alpha, \quad j = 1, 2, \ldots, J \\
& \rho \cdot \left(\sum_{i=1}^{I} X_{jk}\right) = \sum_{l=1}^{L} Y_{kl}, \quad k = 1, 2, \ldots, K \\
& \sum_{i=1}^{I} Y_{ij} = 1, \quad i = 1, 2, \ldots, I \\
& \sum_{i=1}^{I} Y_{ij} \leq M \cdot Y_{j}, \quad j = 1, 2, \ldots, J \\
& \sum_{j=1}^{J} X_{jk} \leq V_{k} G_{k}, \quad k = 1, 2, \ldots, K \\
& \sum_{k=1}^{K} Y_{kl} \leq M_{l} S_{l}, \quad l = 1, 2, \ldots, L \\
& d_{ij} P_{ij} \leq l, \quad i = 1, 2, \ldots, I, \quad j = 1, 2, \ldots, J \\
& q \leq \sum_{j=1}^{J} Q_{j} \leq Q \\
& r \leq \sum_{k=1}^{K} R_{k} \leq R \\
& s \leq \sum_{l=1}^{L} S_{l} \leq S \\
& X_{jk}, Y_{kl} \geq 0, \quad P_{ij}, Q_{j}, R_{k}, S_{l} \in \{0, 1\}.
\end{align*}
\]

In this model, \(\alpha\) is a predetermined confidence levels set by decision makers, and \(C_0\) is a preset constant representing the feasible cost accepted by decision makers. The first constraint implies that uncertainty measure of not returning used products returned by consumers is less than or equal to the given confident level.

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